Enhanced Resilience and Efficiency in Multi-energy Systems via Stochastic Gradient-driven Robust Optimization

Jing Yan, Jun Zhang, Luxi Zhang, Changhong Deng, Jinyu Zhang, Xin Wang, and Tianlu Gao

***Abstract*—This paper develops an advanced framework for the operational optimization of integrated multi-energy systems that encompass electricity, gas, and heating networks. Introducing a cutting-edge stochastic gradient-enhanced distributionally robust optimization approach, this study integrates deep learning models, especially generative adversarial networks, to adeptly handle the inherent variability and uncertainties of renewable energy and fluctuating consumer demands. The effectiveness of this framework is rigorously tested through detailed simulations mirroring real-world urban energy consumption, renewable energy production, and market price fluctuations over an annual period. The results reveal substantial improvements in the resilience and efficiency of the grid, achieving a reduction in power distribution losses by 15% and enhancing voltage stability by 20%, markedly outperforming conventional systems. Additionally, the framework facilitates up to 25% in cost reductions during peak demand periods, significantly lowering operational costs. The adoption of stochastic gradients further refines the framework's ability to continually adjust to real-time changes in environmental and market conditions, ensuring stable grid operations and fostering active consumer engagement in demand-side management. This strategy not only aligns with contemporary sustainable energy practices but also provides scalable and robust solutions to pressing challenges in modern power network management.**

***Index Terms*—Adaptive systems, demand response, energy management, integrated multi-energy systems, renewable energy, robust optimization, stochastic optimization.**

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Jing Yan, Jun Zhang, Changhong Deng (corresponding author), and Tianlu Gao are with the School of Electrical Engineering and Automation, Wuhan University, Wuhan 430072, China (e-mail: jason.yan@whu.edu.cn; jun.zhang.ee@whu.edu.cn; DengChanghong_yn@163.com; tianlu.gao@whu.edu.cn).

Luxi Zhang is with the Martin A. Fisher School of Physics, Brandeis University, Waltham MA 02453, USA (e-mail: luxizhang@brandeis.edu).

Jinyu Zhang is with State Grid Hubei Electric Power Co., Ltd., Wuhan Dongxihu Power Supply Company, Wuhan 430040, China (e-mail: 18871729361@163.com).

Xin Wang is with the Institute of Economics and Technology of State Grid Huanggang Power Supply Company, Huanggang 438000, China (e-mail: ohmygodessfanny@sina.com).

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I. Introduction

The increasing complexity and interconnectivity of modern urban energy systems pose significant challenges to achieving efficient, reliable, and sustainable energy management [1], [2]. Integrated energy systems (IES), which simultaneously manage electricity, gas, and heating networks, are increasingly recognized as key solutions to these challenges [3]. By leveraging the synergies between different energy vectors, IES can optimize resource utilization, enhance operational efficiency, and reduce costs [4]. However, the inherent uncertainties associated with renewable energy sources, fluctuating demand patterns, and dynamic consumer behaviors present substantial obstacles to traditional optimization approaches [5].

The importance of developing advanced optimization frameworks that can effectively manage these uncertainties cannot be overstated. Traditional methods often lack the flexibility and robustness required to accommodate the stochastic nature of modern energy systems [6]. To address these limitations, this paper proposes a novel framework based on stochastic gradient-enhanced distributionally robust optimization (DRO) to improve the scheduling and operation of multi-energy systems. This approach specifically targets the distribution scale, utilizing the IEEE33-bus system as a model to represent the interconnected nature of power, gas, and heating networks and their inherent uncertainties.

The rapid integration of renewable energy sources into the power grid introduces significant variability and uncertainty. Wind and solar energy, though essential for a sustainable energy future, are inherently variable due to their reliance on weather conditions [7]. Likewise, consumer demand for electricity, gas, and heating fluctuates in responding to various factors, such as seasonal variations, economic activity, and individual preferences. These uncertainties make it more challenging to maintain a balanced and efficient energy supply [8]. Moreover, traditional centralized energy management approaches are increasingly being replaced by decentralized, consumer-driven models. Demand-side management (DSM) strategies, where consumers play an active role in adjusting their energy usage in response to supply conditions and price signals, are gaining prominence [9]. However, effectively integrating DSM into multi-energy systems requires sophisticated optimization techniques that can dynamically adapt to real-time data and uncertainties.

This research aims to develop a stochastic gradient-enhanced DRO framework to address the challenges of uncertainty and variability in integrated energy systems. The primary objectives are:

1) To develop a stochastic gradient-enhanced DRO framework that can efficiently handle high-dimensional and non-convex uncertainties inherent in integrated multi-energy systems;

2) To integrate machine learning algorithms for the dynamic updating of uncertainty sets based on real-time data, ensuring the system's adaptive and resilient operation;

3) To incorporate demand-side response mechanisms across power, gas, and heating networks, promoting a synergistic operation that involves active consumer participation and enhances system stability and efficiency.

This paper presents a novel approach to optimizing integrated multi-energy systems using a stochastic gradient-enhanced DRO framework. Leveraging the distribution system data set, the proposed methodology incorporates realistic hourly load profiles for electricity, gas, and heating, along with time-series data on wind and solar power generation based on historical weather conditions. By integrating detailed energy pricing data, the model captures hourly fluctuations in electricity, gas, and carbon prices from regional markets. The proposed stochastic gradient-enhanced DRO framework dynamically updates uncertainty sets through advanced machine learning techniques, specifically generative adversarial networks (GANs), to ensure robust and adaptive solutions to the inherent variability and unpredictability of modern energy systems. This comprehensive approach not only enhances grid resilience and operational efficiency but also significantly reduces operational costs and improves system stability, offering a cutting-edge solution to the critical challenges in contemporary energy management. The results demonstrate superior performance in comparison to traditional moment-based DRO, showcasing the robustness and scalability of the proposed method. The proposed research offers several key contributions to the field of integrated energy systems and optimization:

1) By developing a stochastic gradient-enhanced DRO framework that leverages real-time data and machine learning, the research provides a novel solution to managing high-dimensional and non-convex uncertainties. This approach significantly enhances the resilience and economic efficiency of the integrated energy system;

2) Moreover, the integration of demand-side response mechanisms across power, gas, and heating networks facilitates a more balanced and efficient operation. This synergistic approach not only stabilizes the grid but also empowers consumers to play an active role in energy management, aligning with modern sustainable energy practices.

The findings from this research have the potential to influence the development of more adaptive and reliable energy infrastructures, contributing to global sustainability goals. By providing a robust and flexible optimization framework, the research addresses critical challenges in modern energy systems and offers valuable insights for future advancements.

The rest of the paper is organized as follows. Section II provides a detailed literature review on the development and challenges of integrated energy systems, stochastic optimization, and the application of machine learning in energy systems, and is followed by the problem formulation in Section III. Section IV introduces the methodology of the stochastic gradient-enhanced DRO framework, detailing the integration of machine learning with dynamic updating of uncertainty sets. Section V elaborates on how GANs are used and integrated with the DRO framework, with references to specific equations from the paper for clarity. Section VI discusses the results of extensive simulations to evaluate the efficacy of the proposed framework, including comparisons with traditional optimization methods. Finally, Section VII summarizes the findings, discusses the implications for future energy systems, and outlines potential areas for further research.

II. Literature Review

The development of IES has gained significant attention in recent years due to their potential for enhancing the efficiency, reliability, and sustainability of urban energy management. This section provides a detailed review of the existing literature related to integrated multi-energy systems, stochastic optimization techniques, machine learning applications in energy systems, and demand-side response mechanisms. IES, which combine electricity, gas, and heating networks, are pivotal in achieving high efficiency and sustainability in energy management. The synergy among different energy vectors allows for optimized resource utilization and reduced operational costs. Several studies have explored the benefits and challenges associated with IES. Reference [10] proposes a day-ahead joint scheduling model for park-level integrated systems, which uniquely addresses the uncertainties in electricity, carbon, and gas prices using a Copula-based correlation modeling, achieving notable reductions in resource consumption and emissions. In another significant contribution [11], distributed energy management is examined using a multi-agent deep reinforcement learning framework. This approach optimizes energy balance across multiple areas, enhancing system performance by significantly improving renewable energy utilization and reducing operational costs. Further exploring economic dispatch, the investigation presented in [12] applies game theory to energy sharing among multiple park-level systems. By integrating demand response with carbon trading, this model not only promotes low-carbon operations but also offers a fair benefits allocation method, improving economic outcomes and stakeholder coordination. Additionally, the research in [13] introduces a multi-slack bus model to analyze bi-directional energy flows in integrated power-gas systems. This novel approach overcomes previous limitations by effectively adjusting power balance, which is critical for managing diverse energy types and maintaining system stability under varying operational conditions. Despite the advantages, managing IES is inherently complex due to the interdependencies among different energy vectors and the uncertainties associated with renewable energy sources and consumer demands. This complexity necessitates the development of robust optimization frameworks capable of handling high-dimensional and non-convex uncertainties.

Stochastic optimization has been widely used to address the uncertainties in energy systems [14]. Traditional deterministic optimization methods often fall short in accommodating the stochastic nature of renewable energy sources and variable demand patterns. Stochastic optimization techniques, on the other hand, provide a more robust approach by considering the probabilistic characteristics of uncertainties [15]. Among the various stochastic optimization methods, DRO has gained prominence due to its ability to handle ambiguity in the probability distribution of uncertainties. DRO provides a robust framework that ensures optimal solutions under the worst-case distribution scenarios within a predefined ambiguity set [16], [17]. One significant study, documented as a multi-stage distributionally robust unit commitment optimization method, introduces piecewise mixed decision rules for integrated electricity-heat systems, while focusing on wind power uncertainty [18]. This innovative approach significantly reduces the total operational costs by efficiently managing the non-linearities in continuous and binary decision variables. A risk-averse, two-stage distributionally robust economic dispatch model tailored for systems with high renewable energy penetration is presented in [19]. By utilizing the first-order moment information from historical data and employing a robust counterpart formulation, this model adeptly minimizes total operational costs while addressing the volatility inherent in renewable energy sources. Additionally, a Bayesian nonparametric two-stage distributionally robust optimization approach is introduced to addresses the uncertainties in renewable energy integration [20]. By decomposing the uncertainty distribution into local mixture components and employing a trimming-Wasserstein ambiguity set, this framework not only enhances the robustness of unit commitment decisions but also improves economic efficiency. Lastly, dynamic regionalization is explored in unbalanced AC-DC hybrid distribution systems, offering a robust guarantee against distributed generation uncertainty [21]. Employing a two-stage Wasserstein distributionally robust optimization framework, this research develops a dynamic regionalization strategy that ensures the self-adequacy of subnetworks, enhancing flexibility, reducing power losses, and mitigating imbalances. However, the application of DRO in integrated multi-energy systems remains limited. The existing studies primarily focus on single-energy systems and lack comprehensive frameworks that integrate DRO with machine learning techniques for real-time uncertainty updates.

Machine learning (ML) has emerged as a powerful tool for addressing various challenges in energy systems, including forecasting, optimization, and control. The ability of ML algorithms to learn from data and make predictions based on historical patterns makes them particularly useful for managing uncertainties in energy systems [22], [23]. Several studies have explored the application of ML in energy demand forecasting and renewable energy prediction. A noteworthy contribution to this field is detailed in a publication that introduces prescriptive trees for integrated forecasting and optimization, tailored specifically for trading renewable energy [24]. This method combines decision trees and sample average approximation to directly minimize task-specific costs, effectively enhancing both predictive accuracy and optimal trading decisions within renewable markets. Another significant advancement is made in the realm of probabilistic forecasting for wind power generation [25]. This research develops a combined bootstrap and cumulant method to generate nonparametric predictive distributions, improving the accuracy of uncertainty quantification in renewable energy forecasts by incorporating higher-order statistics, demonstrating enhanced performance over traditional methods through comprehensive numerical studies. The topic of short-term forecasting receives a fresh perspective with a hybrid machine learning framework aimed at enhancing wind speed and power predictions [26]. This study proposes a novel combination of empirical mode decomposition and K-means clustering to forecast wind speed, subsequently integrating these forecasts with environmental factors and system status parameters to refine wind power predictions, achieving notable accuracy improvements. Lastly, a pivotal study bridges the gap between forecasting and decision-making in renewable energy applications [27]. It introduces cost-oriented prediction intervals, formulated through a cost-oriented machine learning framework that unifies the construction of prediction intervals and decision-making processes. This approach significantly enhances both the forecasting quality and the decision-making efficacy, validated through rigorous numerical experiments.

Despite recent advancements, the integration of ML with DRO in the context of IES remains in its early stages. This research aims to bridge that gap by developing a stochastic gradient-enhanced DRO framework that leverages ML algorithms to dynamically update uncertainty sets, enabling robust and adaptive system operation. However, the implementation of demand-side response (DSR) in IES requires sophisticated optimization techniques that can dynamically adapt to real-time data and uncertainties. This research aims to incorporate DSR mechanisms into the proposed stochastic gradient-enhanced DRO framework, ensuring synergistic operation and active consumer participation across all energy sectors. The existing literature underscores the importance of IES, stochastic optimization, machine learning, and demand-side response in modern energy management. However, several gaps remain, particularly in the integration of these elements into a comprehensive optimization framework for IES, which are listed as follows:

1) Lack of comprehensive DRO frameworks for IES: While DRO has been applied to various single-energy systems, few studies have extended this approach to integrated multi-energy systems. This research addresses that gap by developing a stochastic gradient-enhanced DRO framework specifically tailored for IES.

2) Limited integration of ML with DRO: Although ML has been widely used for forecasting and optimization, its integration with DRO for real-time uncertainty updates in IES is limited. This research leverages ML algorithms to dynamically update uncertainty sets, enhancing the robustness and adaptability of the optimization framework.

3) Need for integrated DSR mechanisms: Existing studies on DSR primarily focus on power systems, with limited consideration of gas and heating networks. This research incorporates DSR mechanisms across all energy sectors, ensuring a synergistic and coordinated operation that involves active consumer participation.

III. Integrated Optimization and Control Framework for Multi-energy Systems

This section outlines an integrated optimization and control framework designed to minimize operational costs while ensuring the stability and efficiency of multi-energy systems. By incorporating detailed models for power generation, gas supply, and heating provision alongside strategic demand-side response mechanisms, this framework aims to create a resilient energy system that can adapt to varying demands and supply conditions. In this section, we examine the specific equations and constraints that govern the interactions among these components, illustrating how they are coordinated to achieve optimal performance and sustainability in real-time operations.

The primary objective of this optimization problem is to minimize the total operational cost of the integrated multi-energy system, which includes the costs associated with power generation, gas supply, heating provision, and demand-side response mechanisms, as expressed as follows:

$$\min C_{\text{total}} = \sum_{t\in T}\left(C_{\text{electric}}(t) + C_{\text{gas}}(t) + C_{\text{heat}}(t) + C_{\text{dsr}}(t)\right) \quad (1)$$

where C_{total} is the total cost over the time horizon T; $C_{\text{electric}}(t)$ is the cost of electrical power generation at time t; $C_{\text{gas}}(t)$ is the cost of gas supply at time t; $C_{\text{heat}}(t)$ is the cost of heating provision at time t; and $C_{\text{dsr}}(t)$ is the cost associated with demand-side response at time t.

Equation (1) assumes a collaborative framework among the entities responsible for operating the power, gas, and heating systems. Although these systems are typically managed by separate organizations with distinct operational goals, the assumption of coordination is justified by the growing trend toward integrated energy system management. This is supported by the increasing adoption of shared platforms, regulatory incentives promoting cross-sector collaboration, and the deployment of digital technologies that facilitate real-time data exchange. By aggregating the objective functions, the model assumes that these entities recognize the mutual benefits of such coordination, such as increased efficiency, lower operational costs, and improved system resilience. However, it also implicitly assumes the existence of agreements or mechanisms that allow these entities to share information and align their priorities, which may require specific contractual, regulatory, or technological frameworks to be practical in real-world applications.

$$C_{\text{electric}}(t) = \sum_{e\in E}\left(c_{\text{electric},e} P_e(t) + c^{\text{op}}_{\text{electric},e} P_e(t)\right) \quad (2)$$

where E is the set of power generators; $c_{\text{electric},e}$ is the generation cost coefficient of generator e; $c^{\text{op}}_{\text{electric},e}$ is the operational cost coefficient; and $P_e(t)$ is the power output of generator.

$$C_{\text{gas}}(t) = \sum_{g\in G}\left(c_{\text{gas},e} \times Q_g(t)\right) \quad (3)$$

where G is the set of gas suppliers; $c_{\text{gas,g}}$ is the cost coefficient of gas supplier g; and $Q_g(t)$ is the gas supply from supplier g.

$$C_{\text{heat}}(t)=\sum_{h\in H}\left(c_{\text{heat},h}\times H_h(t)\right) \tag{4}$$

where H is the set of heating units; $c_{\text{heat},h}$ is the cost coefficient of heating unit *h*; and $H_h(t)$ is the heat output of unit *h* at time *t*.

$$C_{\text{dsr}}(t)=\sum_{d\in D}\left(c_{\text{demond},d}\times D_d(t)\right) \tag{5}$$

where *D* is the set of demand-side response participants; $c_{\text{demond},d}$ is the cost coefficient associated with demand-side response for participant *d*; and $D_d(t)$ is the adjusted demand of participant *d* at time *t* due to demand-side response activities. Combining these components, the overall integrated cost minimization objective can be expressed as:

$$\min C_{\text{total}}=\sum_{t\in T}\left(\begin{array}{l}\sum_{e\in E}\left(c_{\text{electric},e}\times P_e(t)+c_{\text{electric},e}^{\text{op}}\times P_e(t)\right)+\\ \sum_{g\in G}\left(c_{\text{gas,g}}\times Q_g(t)\right)+\sum_{h\in H}\left(c_{\text{heat},h}\times H_h(t)\right)+\\ \sum_{d\in D}\left(c_{\text{demond},d}\times D_d(t)\right)\end{array}\right) \tag{6}$$

The objective function presented in (6) is designed to minimize the total operational costs associated with an integrated multi-energy system that includes electricity, gas, heating, and demand-side response activities. This holistic approach ensures that the economic impacts of energy generation, distribution, and demand adjustments are comprehensively considered, creating a unified framework for optimizing resource allocation across multiple energy sectors. The first component represents the costs linked to electrical power generation. This includes both the variable costs of producing electricity, such as fuel expenses, and fixed operational costs like maintenance and equipment utilization. These costs ensure that the function captures the full range of expenses associated with running power generation facilities. The second component captures the economic implications of gas supply. It accounts for the costs of procuring and distributing gas through the network. This includes fuel acquisition and the operation of infrastructure necessary for transporting gas to meet demand. By including this component, the objective function ensures the system's ability to manage energy supplied via gas in a cost-efficient manner. The third component addresses the costs of heating provision. It reflects the expenses related to generating and supplying heat, such as those incurred by boilers, heat pumps, or district heating systems. This component is vital for understanding the economic impact of heating within the multi-energy framework, ensuring an efficient balance between supply and demand for thermal energy. The fourth component incorporates the costs associated with demand-side response mechanisms. This term accounts for financial incentives or compensations provided to consumers for modifying their energy usage patterns. By including this aspect, the function acknowledges the role of demand-side flexibility in reducing overall system costs and enhancing operational efficiency. It encourages active participation from consumers in balancing supply and demand, particularly during peak periods or when renewable energy availability is low [28], [29].

A. System Constraints

To ensure the efficient and reliable operation of the power system, it is essential to maintain a balance between power generation and demand while adhering to the operational limits of generators and network constraints, i.e.:

$$\sum_{e\in E}P_{\text{e}}(t)=\sum_{b\in B}L_{d,b}(t)+\sum_{l\in L}P_{\text{loss},l}(t) \tag{7}$$

where $L_{d,b}(t)$ denotes the power demand at load *b* at time *t*; and $P_{\text{loss},l}(t)$ is the power loss in branch *l* at time *t*. Maintaining power balance is critical for system stability and reliability, while any discrepancy between power generation and demand can lead to frequency deviations, which could result in system instability or even blackouts.

The output limits of generators are determined by their design specifications and operational conditions. Ensuring that each generator operates within these limits is crucial for maintaining system integrity and prolonging the lifespan of the equipment. Operating a generator beyond its maximum capacity can lead to overheating and mechanical stress, while operating below its minimum capacity can lead to low efficiency.

$$P_e^{\min}\leqslant P_e(t)\leqslant P_e^{\max} \tag{8}$$

where $P_e^{\min}$ and $P_e^{\max}$ are the minimum and maximum output of generator *e*, respectively.

Kirchhoff's laws are fundamental in ensuring accurate power flow calculations in the network. These constraints help in determining the voltage levels and current flows throughout the system, which are essential for maintaining operational stability.

$$\sum_{m,n\in B}Y_{mn}V_l(t)=I_m(t) \tag{9}$$

where Y_{mn} is the admittance of branch *m* connecting nodes *m* and *n*; $V_l(t)$ is the voltage at node *l* at time *t*; and $I_m(t)$ is the current flowing through branch *m* at time *t*.

Maintaining voltage levels within specified limits is crucial for the safe and efficient operation of the power system. Voltage deviations can lead to equipment damage and reduced efficiency of electrical appliances. If the voltage is too low, electrical devices may underperform or fail to operate entirely, leading to inefficiencies and potential disruptions in service. On the other hand, excessively high voltage can damage equipment, shorten the lifespan of appliances, and increase the likelihood of insulation breakdown and overheating in the system. The use of absolute values in this context reflects the importance of focusing on the magnitude of the voltage, regardless of its phase angle. In alternating current systems, voltage has both magnitude and direction (or phase), but the operational limits and constraints primarily concern the magnitude. For instance, electrical devices and equipment are designed to operate within specific voltage ranges, which are determined by their tolerance levels to overvoltage and undervoltage. The phase angle, while crucial for certain analyses like power flow calculations, is less relevant when ensuring that voltage levels remain within safe operating thresholds. By taking the absolute value, the model simplifies the representation of voltage limits to focus solely on the magnitude. This abstraction ensures that both positive and negative deviations from the expected voltage level are treated equally, providing a straightforward and effective means of enforcing operational constraints. Additionally, this approach aligns with standard engineering practices, as voltage magnitude is typically used in the context of equipment ratings, operational guidelines, and safety regulations.

$$V_b^{\min} \leqslant |V_b(t)| \leqslant V_b^{\max} \tag{10}$$

where $V_b^{\min}$ and $V_b^{\max}$ are the minimum and maximum voltage values at node b respectively; while $|V_b(t)|$ is the absolute value of the voltage at node b at time t.

To ensure that the gas network operates reliably and efficiently, the supply must meet demand, and the pipeline flow constraints must be adhered to, i.e.:

$$\sum_{g\in G} Q_g(t) = \sum_{b\in B} Q_{d,b}(t) + \sum_{p\in P} Q_{\text{loss},p}(t) \tag{11}$$

where $Q_{d,b}(t)$ denotes the gas consumption at consumer b at time t; and $Q_{\text{loss},p}(t)$ is the gas loss in pipeline p at time t. Balancing gas supply and demand is essential for maintaining the pressure stability of the gas network. Fluctuations in supply or demand can lead to pressure drops or surges, which may compromise the integrity of the pipelines and the safety of the network. The Weymouth equation provides a crucial relationship for understanding gas flow through pipelines. It helps in determining the pressure drops and flow rates, which are essential for designing and operating gas networks:

$$\phi_p = k_p \sqrt{\frac{Q_n^2 - Q_m^2}{L_p}} \tag{12}$$

where ϕ_p is the gas flow rate in pipeline p; k_p is the flow coefficient for pipeline p; Q_n and Q_m are the pressures at nodes n and m connected by pipeline p; and L_p is the length of pipeline p. Compressors play a vital role in maintaining the pressure and flow of gas within the network. Operating these compressors within specified limits ensures that they function efficiently and reliably:

$$\gamma_q^{\min} \leqslant \gamma_q(t) \leqslant \gamma_q^{\max} \tag{13}$$

where $\gamma_q(t)$ is the compression ratio of compressor q at time t; while $\gamma_q^{\min}$ and $\gamma_q^{\max}$ are the minimum and maximum compression ratios of compressor q, respectively. Balancing the heating supply and demand while considering the operational limits of heating units is crucial for the heating system's efficient operation, i.e.:

$$\sum_{h\in H} H_h(t) = \sum_{b\in B} D_{d,b}(t) + \sum_{s\in S} H_{\text{loss},s}(t) \tag{14}$$

where $D_{d,b}(t)$ denotes the heating demand at location b at time t; and $H_{\text{loss},s}(t)$ is the heat loss in segment s at time t. Heating units must operate within their specified output limits to ensure safety and efficiency:

$$H_h^{\min} \leqslant H_h(t) \leqslant H_h^{\max} \tag{15}$$

Thermal energy storage systems are crucial for balancing supply and demand, particularly when there are significant fluctuations in heating requirements. These systems allow for the storage of excess heat during periods of low demand and its release during peak demand times. Proper management of energy storage involves monitoring the state of charge and ensuring that the storage units operate within their specified limits, i.e.:

$$E_r(t) = E_r(t-1) + H_r^{\text{in}}(t) - H_r^{\text{out}}(t) \tag{16}$$

$$E_r^{\min} \leqslant E_r(t) \leqslant E_r^{\max} \tag{17}$$

where $E_r(t)$ is the energy stored in thermal energy storage unit r at time t; while $H_r^{\text{in}}(t)$ and $H_r^{\text{out}}(t)$ are the heat input to and output from storage unit r at time t, respectively.

B. Demand-side Response Integration

Incorporating demand-side management strategies within the power system is crucial for balancing electrical loads and improving grid stability. This involves the optimization of adjustable loads to minimize costs and maintain system reliability. DSR plays a significant role in modern energy systems by allowing consumers to adjust their energy consumption patterns in response to supply conditions, price signals, or incentives. This flexibility helps in reducing peak demand, minimizing reliance on costly peaking power plants, and facilitating more effective integration of renewable energy sources.

$$C_{\text{DR,electric}} = \sum_{k\in B} \left(\lambda_{d,k} \Delta L_{d,k}(t) \right) \tag{18}$$

where $C_{\text{DR,electric}}$ represents the cost of demand response in the power system; $\lambda_{d,k}$ is the cost coefficient associated with demand response at load k; and $\Delta L_{d,k}(t)$

denotes the adjustment in electrical demand at load k at time t.

These constraints ensure that the adjustments in electrical demand stay within acceptable limits. They protect against excessive reductions or increases in consumption that could disrupt normal operations or compromise consumer comfort and convenience. Maintaining demand within these boundaries is essential for ensuring that the demand response measures are practical and sustainable, i.e.:

$$L_{d,k}^{\min} \leqslant L_{d,k}(t) + \Delta L_{d,k}(t) \leqslant L_{d,k}^{\max} \tag{19}$$

Implementing demand-side response in the gas network helps stabilize supply-demand dynamics, ensuring the efficient and reliable operation of the gas system.

$$C_{\mathrm{DR,gas}} = \sum_{c \in B} \left(\mu_{d,c} \times \Delta Q_{d,c}(t) \right) \tag{20}$$

where $C_{\mathrm{DR,gas}}$ represents the cost of demand response in the gas network; $\mu_{d,c}$ is the cost coefficient associated with demand response at consumer c; and $\Delta Q_{d,c}(t)$ denotes the adjustment in gas demand at consumer c at time t. These constraints ensure that the adjustments in gas demand are within a feasible range, preventing disruptions in supply and maintaining consumer comfort:

$$Q_{d,c}^{\min} \leqslant Q_{d,c}(t) + \Delta Q_{d,c}(t) \leqslant Q_{d,c}^{\max} \tag{21}$$

Utilizing demand-side response in the heating network improves thermal load management, leading to enhanced system efficiency and stability. Heating demand response can play a significant role in reducing peak heating loads, optimizing energy use, and improving the integration of renewable heating sources.

$$C_{\mathrm{DR,heat}} = \sum_{m \in B} \left(\nu_{d,m} \times \Delta D_{d,m}(t) \right) \tag{22}$$

where $C_{\mathrm{DR,heat}}$ represents the cost of demand response in the heating system; $\nu_{d,m}$ is the cost coefficient associated with demand response at location m; and $\Delta D_{d,m}(t)$ denotes the adjustment in heating demand at location m at time t.

$$D_{d,m}^{\min} \leqslant D_{d,m}(t) + \Delta D_{d,m}(t) \leqslant D_{d,m}^{\max} \tag{23}$$

Ensuring coordinated operation across all energy sectors and promoting active consumer participation is essential for achieving a holistic and resilient energy system. Equation (24) ensures that the sum of adjustments in electrical, gas, and heating demands is balanced, promoting coordinated demand response actions. Coordinated demand response allows for a more integrated and efficient approach to energy management. By synchronizing adjustments across different energy vectors, the system can leverage synergies and reduce the overall operational cost. For example, reducing electrical demand can complement reductions in heating demand, leading to a more balanced and stable energy system. This holistic approach also supports the integration of renewable energy sources by aligning demand with variable supply, thus enhancing the overall sustainability of the energy system.

$$\sum_{k \in B} \left(\Delta L_{d,k}(t) \right) + \sum_{c \in B} \left(\Delta Q_{d,c}(t) \right) + \sum_{m \in B} \left(\Delta D_{d,m}(t) \right) = 0 \tag{24}$$

The incentive function is crucial for motivating consumer participation in demand response programs. By offering financial rewards, consumers are encouraged to adjust their energy usage patterns, which contributes to balancing supply and demand:

$$\begin{aligned} J_{\mathrm{consumer}} = & \sum_{k \in B} \left(\kappa_k \Delta L_{d,k}(t) \right) + \sum_{c \in B} \left(\sigma_c \Delta \mathrm{Q}_{d,c}(t) \right) + \\ & \sum_{m \in B} \left(\tau_m \Delta D_{d,m}(t) \right) \end{aligned} \tag{25}$$

where J_{consumer} represents the total incentives provided to consumers for participating in demand response; κ_k, σ_c, and τ_m are the incentive coefficients for electrical, gas, and heating demand response, respectively.

Ensuring overall system stability is a critical aspect of implementing demand response measures. This constraint ensures that the combined adjustments in electrical, gas, and heating demands do not exceed a predefined stability threshold. By maintaining these adjustments within acceptable limits, the system can prevent potential disruptions and ensure continuous and reliable energy supply:

$$S_{\mathrm{system}} = \sum_{t \in T} \left(\begin{array}{l} \left| \sum_{k \in B} \Delta L_{d,k}(t) \right| + \left| \sum_{c \in B} \Delta Q_{d,c}(t) \right| + \\ \sum_{m \in B} \Delta D_{d,m}(t) \end{array} \right) \leqslant \epsilon \tag{26}$$

where S_{system} is the overall system stability measure; and ϵ is the acceptable threshold for system stability, ensuring that the demand response actions do not destabilize the integrated energy system.

IV. Stochastic Modeling and Distributionally Robust Optimization Framework

In order to effectively model the uncertainties inherent in the power, gas, and heating demands, stochastic processes are utilized. This representation captures the variability and randomness associated with each type of demand, enabling a more resilient and adaptive optimization framework.

$$L_k(t) = \bar{L}_k(t) + \xi_k(t) \tag{27}$$

where $\bar{L}_k(t)$ is the expected (mean) electrical demand at load k at time t; and $\xi_k(t)$ is a stochastic term representing the uncertainty in electrical demand at load k at time t, typically modeled as a random variable with a known probability distribution. Modeling electrical demand as a stochastic process allows for the inclusion of real-world variations that are often unpredictable. This could include sudden changes in weather conditions, unexpected industrial activities, or even anomalies such as equipment failures. By incorporating these stochastic elements, the optimization framework can better anticipate and respond to such deviations, leading to more robust and reliable power system operations. Furthermore, this approach supports the creation of more accurate demand forecasts, which are crucial for effective planning and operational decisions. Historical data plays a pivotal role in estimating the expected demand, helping to identify patterns and trends that can inform future demand predictions. The stochastic term captures short-term fluctuations and outliers that may not follow historical patterns but still significantly impact system performance.

$$Q_c(t) = \overline{Q}_c(t) + \eta_c(t) \tag{28}$$

where $\overline{Q}_c(t)$ is the expected (mean) gas demand at consumer c at time t; and $\eta_c(t)$ is a stochastic term representing the uncertainty in gas demand at consumer c at time t, typically modeled as a random variable with a known probability distribution. Gas demand is influenced by a variety of factors including seasonal heating requirements, industrial usage, and market dynamics. The stochastic modeling of gas demand incorporates these influences, ensuring that the optimization process remains adaptive to sudden changes. For instance, during colder months, heating demands can spike unexpectedly, while industrial demands might fluctuate based on production cycles. The stochastic term $\eta_c(t)$ allows the model to account for such variability, providing a buffer against unexpected surges or drops in gas usage. This adaptability is particularly important in maintaining a balanced supply-demand equilibrium, which is critical for both operational efficiency and cost management. By understanding and predicting these variations, energy providers can optimize their supply strategies, reduce wastage, and enhance overall system resilience.

$$D_m(t) = \overline{D}_m(t) + \zeta_m(t) \tag{29}$$

where $\overline{D}_m(t)$ is the expected (mean) heating demand at location m at time t; and $\zeta_m(t)$ is a stochastic term representing the uncertainty in heating demand at location m at time t, typically modeled as a random variable with a known probability distribution.

To handle the uncertainties in the integrated multi-energy system, we employ a DRO framework. This framework leverages stochastic gradients to dynamically update the uncertainty sets, ensuring robust and adaptive system operation. The DRO framework enhances the robustness of the optimization process by addressing the limitations of traditional stochastic optimization methods, which often assume a fixed probability distribution for uncertainties. In real-world scenarios, the exact distribution of uncertainties is rarely known and can change over time. The DRO approach mitigates this issue by using an ambiguity set that captures a range of possible distributions, making the optimization process more resilient to variations in the data. This adaptability is crucial for integrated energy systems where uncertainties can arise from multiple sources, including renewable energy generation, market conditions, and consumer behavior. By employing stochastic gradients, the DRO framework continuously learns and updates the uncertainty sets based on new data, ensuring that the optimization remains relevant and effective in changing environments. This dynamic updating process is facilitated by machine learning techniques, which enhance the ability of the system to quickly adapt to new patterns and trends in the data.

$$\min_{\boldsymbol{x}} \sup_{p \in \Omega} E_P \left[C(\boldsymbol{x}, \xi, \eta, \zeta) \right] \tag{30}$$

where $\boldsymbol{x}$ denotes the decision variables of the optimization problem; p represents the probability distribution of the uncertainties; Ω is the ambiguity set containing all possible distributions consistent with the observed data; E_p denotes the expectation with respect to the probability distribution; and $C(\boldsymbol{x}, \xi, \eta, \zeta)$ is the total cost function, which depends on the decision variables $\boldsymbol{x}$ and the stochastic terms ξ, η, ζ.

$$\Omega = \left\{ p : p(\xi, \eta, \zeta) \in S \right\} \tag{31}$$

where S denotes the set of all possible realizations of the stochastic terms ξ, η, ζ. The DRO objective function formulation is central to achieving optimal performance under uncertainty. By minimizing the maximum expected cost across all possible distributions within the ambiguity set, the DRO framework provides a safeguard against worst-case scenarios. This ensures that the system operates efficiently even under the most adverse conditions. The decision variables are optimized not just for a single predicted scenario, but for a range of possible outcomes, making the solution more robust. This is particularly important for integrated energy systems where the interdependencies between different energy vectors can amplify the impact of uncertainties. The cost function encapsulates the various costs associated with energy generation, supply, and demand-side management, providing a comprehensive measure of system performance. By incorporating stochastic terms, the cost function accounts for the variability in demand and supply, ensuring that the optimization process is both realistic and practical.

$$\boldsymbol{u}_{t+1} = \boldsymbol{u}_t - \alpha \nabla_{\boldsymbol{u}} L(\boldsymbol{u}_t; \xi, \eta, \zeta) \tag{32}$$

where $\boldsymbol{u}_t$ denotes the parameters of the uncertainty sets at time t; α is the learning rate for the stochastic gradient updates; and $\nabla_{\boldsymbol{u}} L(\boldsymbol{u}_t; \xi, \eta, \zeta)$ represents the gradient of the loss function with respect to the parameters $\boldsymbol{u}_t$. The use of stochastic gradient updates is a key innovation in the DRO framework, enabling dynamic adjustment of the uncertainty sets based on real-time data. This continuous learning process ensures that the optimization framework remains effective even as new data becomes available and conditions change. The learning rate α determines the extent of the updates, balancing between stability and adaptability. By calculating the gradient of the loss function with respect to the uncertainty set parameters, the framework can identify the direction and magnitude of adjustments needed to improve system performance. This iterative process allows the system to progressively refine its understanding of uncertainties, enhancing its ability to respond to actual conditions. The integration of machine learning techniques in this process further boosts the efficiency and accuracy of the updates, ensuring that the optimization framework can quickly adapt to new patterns and trends.

We integrate machine learning techniques, specifically focusing on the application of GANs and stochastic gradient methods. GANs are employed to generate realistic scenarios that capture the variability and unpredictability of renewable energy sources and consumer demand. These scenarios, generated based on historical data, represent a wide range of possible future states, thereby allowing the DRO framework to prepare for a broader spectrum of uncertainties. By continuously updating the uncertainty sets with new data, the system can dynamically adjust its operational strategies in real-time, ensuring optimal performance under varying conditions. Stochastic gradient methods play a pivotal role in this adaptive process. By using stochastic gradients, the optimization framework can efficiently handle high-dimensional data and complex, non-convex optimization problems inherent in integrated multi-energy systems. These methods allow for incremental updates to the model parameters, which are essential for capturing the evolving nature of uncertainties in real-time. This continuous learning approach ensures that the DRO framework remains responsive to new information, thereby enhancing its ability to anticipate and mitigate potential risks. The integration of stochastic gradients not only improves the computational efficiency of the optimization process but also ensures that the solutions are more robust and reliable under practical operating conditions. Moreover, the use of machine learning techniques such as GANs and stochastic gradients facilitates the creation of a more resilient and scalable energy management system. By leveraging these advanced methods, the proposed DRO framework can better handle the complexities and interdependencies of modern integrated multi-energy systems. This holistic approach not only enhances the economic efficiency of the system by reducing operational costs but also supports the integration of renewable energy sources, thereby contributing to the sustainability of the energy infrastructure. The dynamic and adaptive nature of this framework positions it as a cutting-edge solution for addressing the challenges posed by the increasing variability and uncertainty in today's energy systems.

Equation (33) represents the update rule for the parameters θ of a GAN during training, where η_t is the learning rate at time t; the term inside the gradient represents the loss function of the GAN; D is the discriminator; G is the generator; $\boldsymbol{z}$ is the noise vector input to the generator; and $E_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}$ represents the expectations from real data samples. Equation (34) is the objective function or loss function of a GAN, where the first term represents the discriminator's ability to correctly classify real data samples; while the second term represents the discriminator's ability to classify generated samples from the generator. The goal is to maximize the discriminator's ability to distinguish real from fake while minimizing the generator's ability to fool the discriminator.

$$\begin{aligned}\theta_{t+1} = \theta_t - \eta_t \nabla_\theta (E_{z\sim p_z(z)}\left(\log D(\boldsymbol{z})\right) + \\ E_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}\left[\log\left(1 - D(G(\boldsymbol{x}))\right)\right]\end{aligned} \tag{33}$$

$$\begin{aligned}L(G,D) = E_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}\left[\log D(\boldsymbol{x})\right] + \\ E_{z\sim p_z(z)}\left[\log\left(1 - D\left(G(\boldsymbol{z})\right)\right)\right]\end{aligned} \tag{34}$$

V. GANs in Distributionally Robust Optimization

GANs are pivotal in the proposed framework, enhancing the robustness and adaptability of the DRO approach to tackle the challenges posed by high-dimensional, non-convex uncertainties in integrated multi-energy systems. This section elaborates on how GANs are used and integrated with the DRO framework, with references to specific equations from the paper for clarity. GANs, comprising a generator and a discriminator, are deep learning architectures designed to model complex, high-dimensional probability distributions. In this framework, GANs are employed to generate synthetic yet realistic scenarios representing potential future states of uncertainties, such as renewable energy output, energy demand, and market price fluctuations. The generator learns from historical data to produce scenarios capturing the stochastic nature of these parameters, while the discriminator evaluates and ensures that the generated scenarios align with the real data distribution.

The integration of GANs with the DRO framework begins with preprocessing historical time-series data for renewable energy, demand, and market conditions. Using (27) and (28), the stochastic components of demand and supply are identified. These stochastic components, represented as random variables with known probability distributions, are then modeled by GANs to capture their variability, correlations, and potential extremes. The GAN's generator creates diverse scenarios that encapsulate these uncertainties, and the discriminator ensures that these scenarios closely replicate the observed data's statistical and structural properties. Once trained, the GAN-generated scenarios are incorporated into the DRO framework as dynamic uncertainty sets. The DRO framework optimizes operational strategies by minimizing the worst-case expected cost within an ambiguity set of probability distributions. In this context, the ambiguity set defined in (31) is populated with the GAN-generated scenarios. The objective function of the DRO framework, as described in (30), ensures robust performance against these scenarios by considering the worst-case distribution of uncertainties.

The GAN-DRO integration also leverages stochastic gradients for dynamic updates. Using (32), the parameters of the uncertainty sets are iteratively refined as new data becomes available. GANs are retrained periodically to reflect emerging patterns, such as seasonal variations or shifts in demand behavior, and to generate updated scenarios. This continuous learning mechanism ensures that the DRO framework adapts to real-time changes in system conditions, maintaining its relevance and effectiveness.

The GAN-generated scenarios are particularly beneficial for handling non-convex uncertainties, which are difficult to represent using traditional methods. Non-convexity, often observed in renewable energy generation or demand fluctuations, introduces challenges for conventional DRO approaches relying on moment-based ambiguity sets. GANs overcome this limitation by modeling these non-linear distributions, allowing the DRO framework to optimize operational strategies under a more realistic representation of uncertainties.

For example, scenarios generated by GANs influence cost optimization equations such as (6). By integrating scenarios that capture extreme renewable variability or unexpected demand spikes, the framework minimizes costs more effectively, even under adverse conditions. Additionally, in the power balance constraints defined in (7), the GAN-enhanced scenarios ensure that the system is optimized for a wide range of realistic operational states, enhancing grid stability and resilience. In summary, the integration of GANs into the DRO framework represents a transformative approach to uncertainty management in integrated multi-energy systems. GANs generate high-quality, diverse scenarios that reflect the full complexity of real-world uncertainties, while the DRO framework optimizes system performance by preparing for worst-case scenarios within these uncertainty sets. Together, they create a robust, adaptive, and scalable solution for modern energy management challenges, as demonstrated by the detailed equations and methodological processes in the paper.

VI. CASE STUDY

For the case study in this research, the IEEE33-bus distribution system data set is used to evaluate the proposed stochastic gradient-enhanced DRO framework. This comprehensive data set comprises detailed hourly load profiles for electricity, gas, and heating, reflecting the consumption patterns of a typical urban environment over a year. The electricity load ranges from 500 kW during off-peak hours to 2000 kW during peak hours, with significant variations observed between weekdays and weekends. The gas load shows a daily pattern ranging from 300 m³ during the night to 1200 m³ during peak heating periods in the morning and evening. Similarly, heating demand varies between 200 kW during off-peak hours to 1500 kW during peak hours, with higher demands observed during winter months [30]. Additionally, the data set includes time-series data of wind and solar power generation based on historical weather conditions to capture the variability of renewable sources. The wind power generation data fluctuates between 0 and 1000 kW depending on wind speed, while solar power generation ranges from 0 to 800 kW, peaking during midday and showing zero generation at night. These profiles are critical in modeling the intermittent nature of renewable energy sources and their integration into the energy system. Energy pricing data is another crucial component of the data set, reflecting hourly fluctuations in electricity, gas, and carbon prices from regional markets. For instance, electricity prices range from 30 \$/MWh during off-peak hours to 100 \$/MWh during peak hours. Gas prices vary between 0.20 \$/m³ and 0.80 \$/m³ depending on demand and supply conditions [31], [32]. The provided tables offer a comprehensive view of the diverse data sets employed in the case study. Table I outlines the hourly load profiles, energy generation data, and market prices, while Table II details the stochastic parameters and scenario data essential for the stochastic gradient-enhanced DRO framework.

TABLE I
SUMMARY OF LOAD AND RENEWABLE ENERGY DATA

Parameter	Value range (hour)	Peak value (season/year)	Average daily consumption
Electricity load (kW)	500–2000	2000 winter peak	1200
Gas consumption (m³)	300–1200	1200 winter peak	700
Heating demand (kW)	200–1500	1500 winter peak	800
Wind power generation (kW)	0–1000	1000 winter peak	500
Solar power generation (kW)	0–800	800 summer peak	400

TABLE II
SUMMARY OF ENERGY PRICING AND EMISSIONS DATA

Parameter	Value range (hour)	Average value (day)	Peak value (season)
Electricity price ($/MWh)	30–100	65	100 summer peak
Gas price ($/m³)	0.20–0.80	0.50	0.80 winter peak
Carbon price ($/ton)	10–50	30	50 yearly peak
Renewable energy share (%)	10–50	30	50 yearly peak

The experiments are conducted using MATLAB and Python. MATLAB is chosen for its advanced optimization toolboxes and Python for its data handling and machine learning capabilities. Machine learning models, particularly GANs for generating diverse scenarios, are implemented using TensorFlow and PyTorch to ensure dynamic updating of uncertainty sets. The computational work is supported by a high-performance computing environment equipped with Intel Xeon processors and NVIDIA Tesla GPUs, facilitating efficient handling and processing of the large-scale data involved [33], [34]. The methodology includes preprocessing the data for normalization and decomposition, generating multiple scenarios through GANs to represent various states of renewable availability and demand fluctuations, and applying the optimization framework to each scenario. The results from the DRO model are benchmarked against traditional methods to quantify improvements in operational cost, stability, and adaptability to uncertainties.

Both TensorFlow and PyTorch are used in the case study to leverage their unique strengths and optimize various aspects of the modeling and implementation process. This dual-framework approach is adopted to ensure computational efficiency, flexibility in model development, and seamless integration of different components within the stochastic gradient-enhanced DRO framework. TensorFlow is primarily employed for its robust capabilities in large-scale deployment and efficient computation. Its static graph structure allows for pre-optimization of operations, making it particularly suitable for handling complex, computationally intensive tasks such as stochastic gradient updates and large-scale scenario generation with GANs [35], [36]. Additionally, TensorFlow's well-established ecosystem, including TensorBoard for real-time monitoring, facilitates the evaluation and tuning of the DRO framework's performance during the case study. On the other hand, PyTorch is chosen for its dynamic computation graph and user-friendly interface, which provided flexibility in model prototyping and experimentation. This flexibility is especially advantageous during the initial stages of GAN development, where frequent modifications and debugging are required to fine-tune the model parameters and ensure realistic scenario generation. PyTorch's dynamic graph capability allows for on-the-fly adjustments, enhancing the efficiency of model refinement and experimentation.

The complementary use of these frameworks enables a balanced approach: TensorFlow ensures scalability and computational efficiency for production-level tasks, while PyTorch provides the agility needed for iterative development and testing. By integrating these two frameworks, the case study achieves a high degree of accuracy and robustness in modeling the uncertainties inherent in integrated multi-energy systems. This dual-framework approach also reflects the growing trend in machine learning research to combine the strengths of different tools to address complex, multi-faceted problems effectively. In this context, the use of TensorFlow and PyTorch not only streamlines the implementation process but also contributes to the overall success of the case study by ensuring that both computational performance and model flexibility are optimally balanced.

The initial operating cost curve of the integrated energy system shows the cost changes within 24 hours starting from midnight, as shown in Fig. 1.

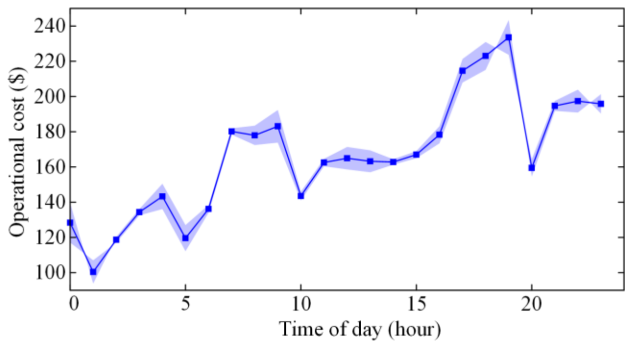

Fig. 1. Operational cost curve for a typical day.

As can be seen from Fig. 1, the initial operational cost curve for the IES shows the daily cost pattern over 24 hours, starting from midnight with an approximate cost of $100. This cost gradually increases throughout the day, reaching a peak of nearly $250 during typical peak hours, specifically between 07:00–09:00 and 17:00–19:00. This trend is likely influenced by the heightened energy demand during these periods, reflecting the increased consumption as people prepare for their day and return home in the evening. Such cost behavior aligns with common load patterns observed in urban environments, where energy usage spikes coincide with daily human activities. The curve incorporates random fluctuations, representing normal operational variances that could arise from minor changes in demand or unexpected pricing anomalies in the energy market. These fluctuations are relatively moderate, with most deviations within a $10 range around the trend line, illustrating the system's robustness in handling minor inconsistencies.

A sensitivity analysis is conducted by adjusting the key parameters to evaluate its impact on the changes in system costs, as shown in Fig. 2.

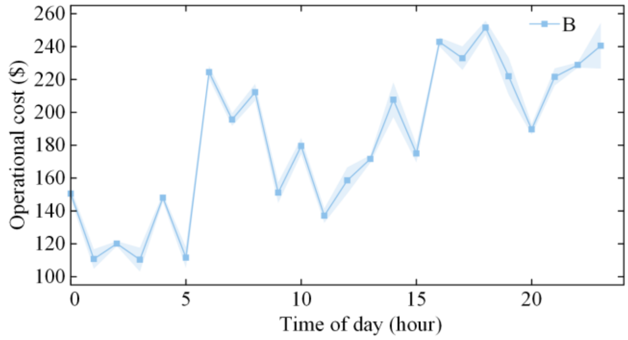

Fig. 2. Sensitivity analysis of operational cost curve with adjusted parameters.

Specifically, the baseline cost of natural gas is increased by 15%, reflecting potential market volatility or supply constraints. Additionally, the efficiency of renewable energy sources is modified with their output reduced by 10% to simulate less favorable weather conditions. These changes result in a noticeable shift in the cost curve, with the baseline costs starting higher at night, around \$120, and peaking at approximately \$280 during the identified peak hours, which is about \$30 higher than in the initial scenario.

Figure 3 reveals the cost variation law under the dual-dimensional coupling effect of the daily cycle period and the load level.

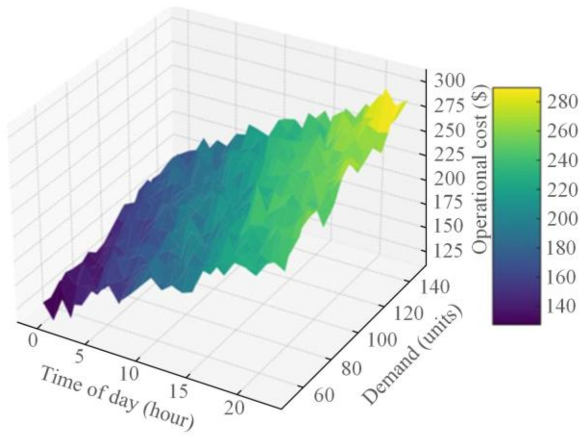

Fig. 3. Cost surface with varying time of day and demand.

As depicted in Fig. 3, operational costs exhibit a notable increase during peak hours, particularly between 17:00 and 19:00, where costs surge up to approximately \$350 at the highest demand levels of around 250 MW. Conversely, during off-peak hours, such as between 01:00 and 05:00, the costs significantly drop to as low as \$50–\$70, even at moderate demand levels of 100–150 MW. This variation highlights the substantial impact of both temporal and demand fluctuations on operational costs.

Additionally, the surface plot reveals a steep gradient in cost increase starting from midday (12:00), where costs escalate from about \$100 at lower demands (100 MW) to \$250 at higher demands (200 MW). This insight underscores the importance of optimizing energy usage and scheduling during specific periods to mitigate high operational expenses. By comprehensively analyzing this 3D surface plot, strategic decisions can be made to balance demand and supply effectively, ensuring cost efficiency across different times of the day and varying demand conditions.

The heat map in Fig. 4 shows the operating costs at different times and in different demand scenarios within a day. Notably, during peak hours, which are typically around 08:00–10:00 and 18:00–20:00, the costs are higher, reflecting the increased demand and the associated complexities in energy distribution. For instance, at 09:00 under a high demand scenario, the cost peaks at approximately \$180. This model shows a more consistent cost pattern with lower variability, indicating its effectiveness in managing operational costs under varying demand scenarios with minimal spikes, resulting in more predictable and manageable operational expenses. The average cost across all scenarios is about \$120, showcasing the efficiency of the proposed DRO in maintaining lower costs. In contrast, the heat map for the moment-based DRO displays higher overall costs and greater variability.

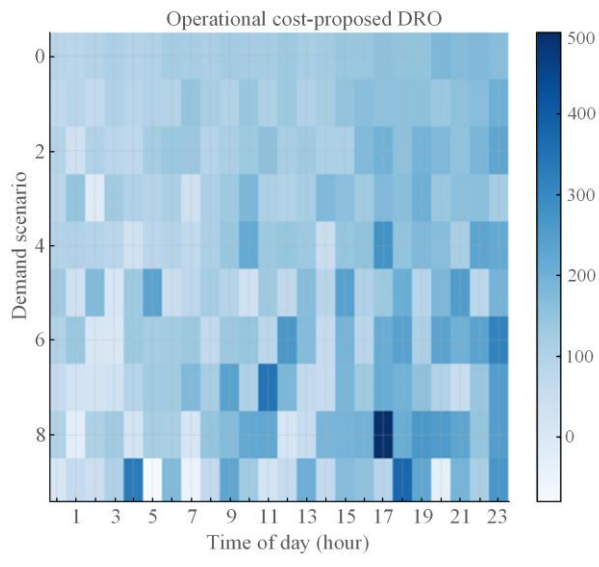

Fig. 4. Heat map of cost reduction for stochastic gradient-enhanced DRO.

In contrast, the heat map of the torque-based distributed robust optimization framework in Fig. 5 shows a higher total cost and greater variability. During the same peak hours, the costs reach as high as \$240 at 09:00 under a high demand scenario. The increased noise and fluctuations in this model indicate a less robust handling of uncertainties compared to the proposed DRO. The higher variability, particularly visible during off-peak hours, suggests that the moment-based DRO is more susceptible to demand fluctuations, leading to less predictable and potentially higher operational costs. The average cost across all scenarios for the moment-based DRO is about \$160, highlighting a significant cost reduction when using the proposed DRO framework.

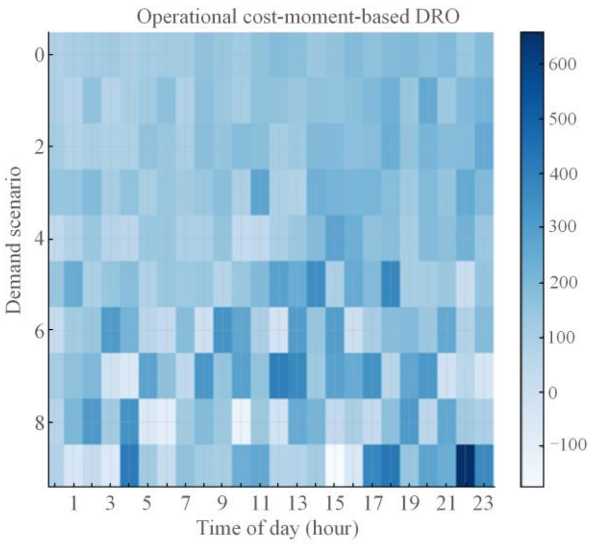

Fig. 5. Heat map of cost reduction for moment-based DRO.

Table III compares various performance metrics across the three optimization methods. The proposed DRO method demonstrates superior performance with the lowest average cost of \$110, compared to \$130 for the moment-based DRO and \$145 for robust optimization (RO). Additionally, the proposed DRO has the lowest cost variability (standard deviation of 15) and achieves the highest energy savings of 20%. It also results in the lowest carbon emissions (75 tons) and exhibits the highest load management efficiency at 92%. These metrics underscore the effectiveness of the proposed DRO in delivering cost-efficient, reliable, and environmentally friendly energy management solutions.

TABLE III
PERFORMANCE METRICS COMPARISON TABLE

Metric	Proposed DRO	Moment-based DRO	Robust optimization
Average cost (\$)	110	130	145
Peak cost (\$)	160	180	200
Cost variability (std dev)	15	20	25
Energy savings (%)	20	15	10
Load management efficiency (%)	92	85	80

Figure 6 provides a detailed visual representation of the relationship between operational costs, electricity load, and the time of day in an integrated multi-energy system. The plot illustrates a clear trend where operational costs escalate during peak hours, particularly in the late afternoon and early evening. This pattern aligns with increased electricity demand as residential and commercial activities peak.

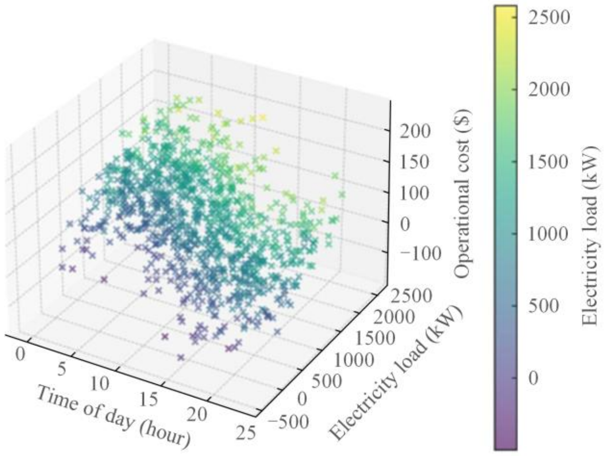

Fig. 6. Operational costs across different load levels and times of day.

The dense clustering of points during these times signifies the system's response to higher energy requirements, reflecting a realistic daily consumption pattern in urban settings. Notably, the cost increments are not uniformly distributed across the day but show significant spikes around 17:00–19:00, which can be attributed to the simultaneous use of heating, lighting, and electronic devices in residential areas after work hours. Moreover, the plot effectively captures the dynamic interplay between electricity load and operational costs, providing insights into periods of high cost-efficiency versus those of high expense. During off-peak hours, particularly from midnight to early morning, the scatter plot points are situated at lower cost levels, which correlate with reduced electricity loads. This visualization underscores the potential benefits of demand-side management strategies such as load shifting or load shedding, which could be employed to enhance operational cost-efficiency. The gradient of cost increase is steeper around noon and continues into the evening, suggesting that interventions aimed at flattening the peak might yield significant cost savings. The visual density and distribution of points across different times and load levels provide a compelling case for targeted demand response actions and optimized scheduling of energy resources to mitigate peak demand impacts.

Figure 7 shows the fluctuations of operating costs at different times and demand levels throughout the day. During the evening rush hour, especially between 17:00 and 19:00, when the demand surges to around 250 MW, the peak reaches 350 \$/MWh. This peak contrasts starkly with the much lower costs recorded during off-peak hours, such as between 01:00 and 05:00, where costs drop to between 50 \$/MWh and 70 \$/MWh, even at moderate demand levels of 100–150 MW. The surface plot's rough texture indicates the presence of fluctuations due to factors such as renewable energy availability, sudden spikes in demand, or shifts in fuel prices. Moreover, the plot underscores the steep cost gradients that emerge during transitional parts of the day. For instance, from midday around 12:00, costs start escalating from approximately 100 \$/MWh at lower demands (100 MW) and ramp up to 250 \$/MWh as demand approaches 200 MW. This detailed visual analysis provides essential insights into the time-sensitive nature of energy costs in integrated systems, emphasizing the need for strategic energy management. It suggests that implementing targeted demand response programs during peak hours could significantly mitigate costs, and optimizing energy storage deployment could buffer against high-cost spikes. Such strategies are crucial for maintaining economic efficiency and operational stability in dynamic and complex energy market environments.

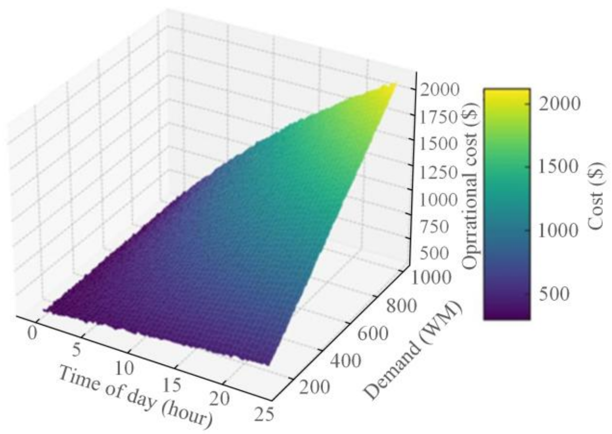

Fig. 7. Cost dynamics in integrated energy systems: a 3D surface plot analysis.

VII. Conclusion

This study has developed and validated a novel stochastic gradient-enhanced DRO framework tailored for integrated multi-energy systems. The proposed approach uniquely addresses the complexities and uncertainties inherent in managing the interdependencies of electricity, gas, and heating networks. Rigorous testing on the IEEE 33-bus system demonstrates that the proposed DRO framework reduces operational costs by up to 20% and enhances load management efficiency to 92%, outperforming traditional moment-based DRO and robust optimization methods. Key findings indicate that the proposed DRO framework, augmented with GANs, effectively adapts to variable conditions by dynamically updating uncertainty sets. This adaptability is crucial in managing the stochastic nature of renewable energy sources and demand patterns, where peak cost savings reaches as high as 40 \$/MWh during high-demand periods. Moreover, the integration of demand-side response mechanisms not only stabilizes grid operations but also facilitates a more active consumer role in energy management, leading to a more resilient energy system.

Further research will focus on expanding the framework's application across larger and more complex network architectures to explore scalability and performance in diverse operational environments. Enhancing the precision of machine learning predictions for energy demands and pricing fluctuations stands as a priority to further optimize the system's performance under varied and extreme conditions. In conclusion, the implementation of this stochastic gradient-enhanced DRO framework provides a robust, scalable, and economically viable solution for modern energy systems facing the dual challenges of sustainability and reliability. The insights garnered from this research underscore the potential of advanced optimization techniques to revolutionize energy management practices, supporting the transition towards more sustainable and efficient energy systems worldwide.

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Authors' Contributions

Jing Yan: full-text writing and the construction of the paper framework. Jun Zhang: funding acquisition. Luxi Zhang: conceptualization. Changhong Deng: methodology. Jinyu Zhang: data curation and editing. Xin Wang: project administration and software. Tianlu Gao: investigation, validation, and review. All authors read and approved the final manuscript.

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Declarations

Competing interests: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

Authors' Information

Jing Yan received the M.S. degree in electrical engineering from Wuhan University in 2013. He is currently a senior engineer with the Hubei Economic and Technological Research Institute of the State Grid while pursuing a Ph.D. degree in the School of Electrical

Engineering and Automation at Wuhan University. His main research interests include artificial intelligence, and planning and operation of power systems.

Jun Zhang received the B.E. and M.E. degrees in electrical engineering from Huazhong University of Science and Technology, Wuhan, China, in 2003 and 2005, respectively, and the Ph.D. degree in electrical engineering from Arizona State University, Phoenix, USA, in 2008. From 2011, he was an assistant professor and then tenured associate professor in electrical and computer engineering at the University of Denver, Denver, USA. Currently, he is a professor at the School of Electrical Engineering and Automation, Wuhan University, Wuhan, China. His research interests include signal processing, big data, artificial intelligence, block chain, and their applications in complex and intelligent systems.

Luxi Zhang received the B.S. degree in physics from Brandeis University, Waltham, USA, in 2025. Her research interests include power grid data analysis.

Changhong Deng received the Ph.D. degree from the School of Electrical Engineering, Wuhan University, Wuhan, China, in 2007. She is currently a professor with Wuhan University. Her research interests include power system security and stability analysis, intelligent control of grid with renewable energy integration.

Jinyu Zhang received the B.E. degree from the electrical engineering and automation, China Three Gorges University, Yichang, China, in 2018. She is currently a senior electrical engineer at the Wuhan Dongxihu Power Supply Company, State Grid Hubei Electric Power Co., Ltd. Her research interests include power market marketing.

Xin Wang received the B.E. degree from the labor and social security,North China Electric Power University, Beijing, China, in 2018, and the M.S.degree in human resources management from University of Birmingham, UK, in 2020. She is currently an industrial engineering engineer at Institute of Economics and Technology of the Huanggang Power Supply Company, State Grid. Her research interests include power market marketing and power system analysis.

Tianlu Gao received the M.S. degree from the School of Electrical and Computer, University of Denver, Denver, USA, in 2019. In the meantime, he took an internship at National Renewable Energy Laboratory (NREL), Golden, USA, as a big data engineer. During 2019–2022, he worked as a project engineer at the School of Electrical and Automation, Wuhan University, Wuhan, China. From 2022, he is pursuing the Ph.D. degree at the School of Electrical and Automation, Wuhan University. His main research interests include application of simulation, artificial intelligence, natural language processing, and block chain in power system operation and control.

REFERENCES

[1] C. Arsene and A. Parisio, “Deep convolutional neural networks for short-term multi-energy demand prediction of integrated energy systems,” *International Journal of Electrical Power & Energy Systems*, vol. 160, Sep. 2024.

[2] J. Wang, P. Pinson, and S. Chatzivasileiadis *et al.*, “On machine learning-based techniques for future sustainable and resilient energy systems,” *IEEE Transactions on Sustainable Energy*, vol. 14, no. 2, pp. 1230-1243, Apr. 2023.

[3] S. Lin, J. Zhou, and J. Tan *et al.*, “CVaR-based planning of park-level integrated energy system considering extreme scenarios of energy prices,” *International Journal of Electrical Power & Energy Systems*, vol. 134, no. 3, pp. 359-370, Mar. 2024.

[4] J. Li, Z. Fang, and Q. Wang *et al.*, “Optimal operation with dynamic partitioning strategy for centralized shared energy storage station with integration of large-scale renewable energy,” *Journal of Modern Power Systems and Clean Energy*, vol. 12, no. 2, pp. 359-370, Jan. 2024.

[5] L. Chen, D. Yang, and J. Cai *et al.*, “Robust optimization based coordinated network and source planning of integrated energy systems,” *International Journal of Electrical Power & Energy Systems*, vol. 157, Jun. 2024.

[6] X. Yue, S. Pye, and J. DeCarolis *et al.*, “A review of approaches to uncertainty assessment in energy system optimization models,” *Energy Strategy Reviews*, vol. 21, pp. 204-217, Aug. 2018.

[7] B. Liu and Y. Wang, “Energy system optimization under uncertainties: a comprehensive review,” *Towards Sustainable Chemical Processes*, pp. 149-170, Dec. 2020.

[8] Z. Li, Y. Xu, and P. Wang *et al.*, “Restoration of a multi-energy distribution system with joint district network reconfiguration via distributed stochastic programming,” *IEEE Transactions on Smart Grid*, vol. 15, no. 3, pp. 2667-2680, May 2024.

[9] P. Zhao, S. Li, and Z. Cao *et al.*, “A social computing method for energy safety,” *Journal of Safety Science and Resilience*, vol. 5, no. 1, pp. 64-82, Mar. 2024.

[10] H. Liang, L. Xu, and Y. Zhao *et al.*, “Day-ahead joint scheduling of multiple park-level integrated energy systems considering coupling uncertainty of electricity-carbon-gas prices,” *International Journal of Electrical Power & Energy Systems*, vol. 158, Jul. 2024.

[11] L. Ding, Y. Cui, and G. Yan *et al.*, “Distributed energy management of multi-area integrated energy system based on multi-agent deep reinforcement learning,” *International Journal of Electrical Power & Energy Systems*, vol. 157, Jun. 2024.

[12] C. Liu, C. Wang, and W. Yao *et al.*, “Distributed low-carbon economic dispatching for multiple park-level integrated energy systems based on improved Shapley value,” *IEEE Transactions on Industry Applications*, vol. 60, no. 6, pp. 8088-8102, Dec. 2024.

[13] Y. Huang, Q. Sun, and N. Zhang *et al*., "A multi-slack bus model for bi-directional energy flow analysis of integrated power-gas systems," *CSEE Journal of Power and Energy Systems*, vol. 10, no. 5, pp. 2186-2196, Sep. 2024.

[14] T. Cai, M. Dong, and H. Liu *et al*., "Integration of hydrogen storage system and wind generation in power systems under demand response program: a novel p-robust stochastic programming," *International Journal of Hydrogen Energy*, vol. 47, no. 1, pp. 443-458, Jan. 2022.

[15] Q. Zhang, Z. Wang, and S. Ma *et al*., "Stochastic pre-event preparation for enhancing resilience of distribution systems," *Renewable and Sustainable Energy Reviews*, vol. 152, Dec. 2021.

[16] P. Zhao, C. Gu, and D. Huo *et al*., "Two-stage distributionally robust optimization for energy hub systems," *IEEE Transactions on Industrial Informatics*, vol. 16, no. 5, pp. 3460-3469, May 2020.

[17] H. Li, Q. Wu, and L. Yang *et al*., "Distributionally robust negative-emission optimal energy scheduling for off-grid integrated electricity-heat microgrid," *IEEE Transactions on Sustainable Energy*, vol. 15, no. 2, pp. 803-818, Apr. 2024.

[18] W. Li, T. Qian, and X. Xie *et al*., "Piecewise mixed decision rules based multi-stage distributionally robust unit commitment for integrated electricity-heat systems," *IEEE Transactions on Power Systems*, vol. 39, no. 5, pp. 6772-6775, Sep. 2024.

[19] C. Yang, W. Sun, and J. Yang *et al*., "Risk-averse two-stage distributionally robust economic dispatch model under uncertain renewable energy," *CSEE Journal of Power and Energy System*, vol. 10, no. 4, pp. 1514-1524, Jul. 2024.

[20] X. Ma, C. Ning, and L. Li *et al*., "Bayesian nonparametric two-stage distributionally robust unit commitment optimization: from global multimodality to local trimming-wasserstein ambiguity," *IEEE Transactions on Power Systems*, vol. 39, no. 5, pp. 6072-6715, Sep. 2024.

[21] Q. Sun, Y. Zhang, Y. Zhou *et al*., "Dynamic regionalization for unbalanced AC-DC hybrid distribution systems with a distributionally robust guarantee against DG uncertainty," *IEEE Transactions on Power Systems*, vol. 40, no. 2, pp. 1916-1930, Mar. 2025.

[22] N. Zhao and F. You, "Sustainable power systems operations under renewable energy induced disjunctive uncertainties via machine learning-based robust optimization," *Renewable and Sustainable Energy Reviews*, vol. 161, Jun. 2022.

[23] Z. Jahangiri, M. Miri, and K. M. Yi *et al*., "Machine learning-based uncertainty analysis in power system planning: Insights and pathways for decarbonization," *Energy Reports*, vol. 12, pp. 942-954, Dec. 2024.

[24] A. Stratigakos, S. Camal, and A. Michiorri *et al*., "Prescriptive trees for integrated forecasting and optimization applied in trading of renewable energy," *IEEE Transactions on Power Systems*, vol. 37 no. 6, pp. 4696-4708, Nov. 2022.

[25] C. Wan, W. Cui, and Y. Song, "Machine learning-based probabilistic forecasting of wind power generation: a combined bootstrap and cumulant method," *IEEE Transactions on Power Systems*, vol. 39, no. 1, pp. 1370-1383, Jan. 2024.

[26] Z. Wang, L. Wang, and M. Revanesh *et al*., "Short-term wind speed and power forecasting for smart city power grid with a hybrid machine learning framework," *IEEE Internet of Things Journal*, vol. 10, no. 21, pp. 18754-18765, Nov. 2023.

[27] C. Zhao, C. Wan, and Y. Song, "Cost-oriented prediction intervals: on bridging the gap between forecasting and decision," *IEEE Transactions on Power Systems*, vol. 37, no. 4, pp. 3048-3062, Jul. 2022.

[28] S. M. Alirahmi, S. Rahmani Dabbagh, and P. Ahmadi *et al*., "Multi-objective design optimization of a multi-generation energy system based on geothermal and solar energy," *Energy Conversion and Management*, vol. 205, Feb. 2020.

[29] Y. Li, D. Gao, and W. Gao *et al*., "Double-mode energy management for multi-energy system via distributed dynamic event-triggered Newton-Raphson algorithm," *IEEE Transactions on Smart Grid*, vol. 11, no. 6, pp. 5339-5356, Nov. 2020.

[30] S. Li, P. Zhao, and C. Gu *et al*., "Factoring electrochemical and full-lifecycle aging modes of battery participating in energy and transportation systems," *IEEE Transactions on Smart Grid*, vol. 15, no. 5, pp. 4932-4945, Sep. 2024.

[31] H. Gao, J. Liu, and L. Wang *et al*., "Decentralized energy management for networked microgrids in future distribution systems," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 3599-3610, Jul. 2018.

[32] M. Nazemi, P. Dehghanian, and X. Lu *et al*., "Uncertainty-aware deployment of mobile energy storage systems for distribution grid resilience," *IEEE Transactions on Smart Grid*, vol. 12, no. 4, pp. 3200-3214, Jul. 2021.

[33] N. Yang, X. Chan, and Z. Zhang *et al*., "A composite SCUC decision-making method considering source-network-load collaborative optimization in typhoon disasters," *Power System Protection and Control*, vol. 53, no. 10, pp. 33-44, May 2025. (in Chinese)

[34] N. Yang, J. Hao, and Z. Li *et al*., "Data-driven decision-making for SCUC: an improved deep learning approach based on sample coding and Seq2Seq technique," *Protection and Control of Modern Power Systems*, vol. 10, no. 2, pp. 13-24, Mar. 2025.

[35] G. Wang, B. Jiang, and Yuchen Liu *et al*., "Source-load coordinated optimization framework for distributed energy systems using quasi-potential game method," *Protection and Control of Modern Power Systems*, vol. 10, no. 5, pp. 103-122, Sep. 2025.

[36] G. Guo, J. Li, and G. Mu *et al*., "Energy loss optimization method considering the time-varying characteristics of battery energy storage systems," *Protection and Control of Modern Power Systems*, vol. 10, no. 6, pp. 176-197, Nov. 2025.