A Consortium Blockchain-enabled Double Auction Mechanism for Peer-to-peer Energy Trading among Prosumers

Shichang Cui, *Member*, *IEEE*, Shuang Xu, Fei Hu, Yong Zhao, Jinyu Wen, *Senior Member*, *IEEE*, and Jinsong Wang

Abstract—**This paper investigates a double auction-based peer-to-peer (P2P) energy trading market for a community of renewable prosumers with private information on reservation price and quantity of energy to be traded. A novel competition padding auction (CPA) mechanism for P2P energy trading is proposed to address the budget deficit problem while holding the advantages of the widely-used Vickrey-Clarke-Groves mechanism. To illustrate the theoretical properties of the CPA mechanism, the sufficient conditions are identified for a truth-telling equilibrium with a budget surplus to exist, while further proving its asymptotical economic efficiency. In addition, the CPA mechanism is implemented through consortium blockchain smart contracts to create safer, faster, and larger P2P energy trading markets. The proposed mechanism is embedded into blockchain consensus protocols for high consensus efficiency, and the budget surplus of the CPA mechanism motivates the prosumers to manage the blockchain. Case studies are carried out to show the effectiveness of the proposed method.**

Index Terms—**Blockchain smart contract, budget, double auction mechanism, economic efficiency, peer-to-peer energy trading, prosumers.**

I. Introduction

The rapid growth of distributed energy resources (DER), such as residential rooftop photovoltaics (PV) and battery energy storage systems (BESS), is transforming energy consumers into prosumers who actively participate in both energy production and consumption [1]. However, traditional centralized power grid management is expensive and impractical for managing DER, whose characteristics differ significantly in terms of diversity, capability, location, and ownership [2]. Establishing localized peer-to-peer (P2P) energy markets, where neighbouring prosumers can directly trade energy with one another, is recognized as a potent measure for the adoption and management of large numbers of small-scale DERs.

The P2P energy trading paradigm has the potential to lower investments in upstream generation capacity, reduce high-voltage transmission requirements, and alleviate uncertainty in the power grid [3]. In addition, the feed-in tariff (FiT) for prosumers to sell surplus generation to the utility grid is always lower than the price for purchasing energy, so that prosumers are incentivized to trade energy with each other before resorting to separate transactions with the utility grid. Moreover, the observed decline in FiTs in several countries including the US, UK, and Australia further motivates prosumers to take part in localized P2P market transactions [4].

Theoretically, the P2P energy trading market can be described as a many-to-many matching market. Many works in literature have designed matching or trading mechanisms to incentivize energy transactions among prosumers. These mechanisms include bilateral contracts based on agent-to-agent negotiation [5]–[7] and profit-sharing mechanisms based on cooperative game theory [8]–[10]. Another category of mechanisms involves double auction mechanisms which allow for competitive bids and asks from prosumers. In this category, some studies assume that prosumers do not behave strategically. For example, an iterative double auction is adopted in [11] where prosumers are price-takers, whereas the authors in [12] consider a continuous double auction and assume that all prosumers use a unified bidding strategy. Reference [13] designs a multi-stage and multi-period double auction based on the assumption that prosumers truthfully report their private information. In practice, the P2P markets are likely to experience considerable loss of economic efficiency when prosumers act strategically or exercise market power. References [14]–[16] explore

Received: October 3, 2023
Accepted: January 10, 2024
Published Online: May 1, 2024
Shuang Xu (corresponding author) is with the State Key Laboratory of Advanced Electromagnetic Engineering and Technology, Huazhong University of Science and Technology, Wuhan 430074, China; and also with the State Grid Hubei Electrical Power Co., Ltd. Economics Research Institute, Wuhan 430077, China (e-mail: xushuang_sgcc@163.com).
DOI: 10.23919/PCMP.2023.000038

ways to eliminate market power, thereby enhancing economic efficiency by encouraging truth-telling among prosumers. However, references [14] and [15] assume a trusted auctioneer to handle the auction process. In addition, a Vickrey-Clarke-Groves (VCG) double auction mechanism is adopted in [16] to maximize the social welfare with a budget deficit, i.e., the total payments of buyer prosumers may be less than the total revenues to seller prosumers. Therefore, the VCG-based P2P market is unsustainable as external subsidies are needed in case of any budget deficiency.

In practice, relying on a trusted auctioneer to manage the process of P2P trading involves a number of risks, such as single points of failure and susceptibility to bribery [17], along with privacy and security concerns. Moreover, should the number of participants continue to grow, the central management and operation of P2P energy trading in near real time would become progressively challenging. These observations encourage the adoption of blockchain technology, as it allows for a decentralized management of data and transactions by a group of peers via consensus protocols [18]. This eliminates the need for a central party along with the data privacy issues it implies. Blockchain technologies allow for automated execution of tamper-proof smart contracts, and thus offer an opportunity for more reliable and cost-efficient P2P energy trading.

The application of blockchain technology in P2P energy trading has drawn considerable attention in the research community. The extant literature can be classified into two categories. The first category considers the application of existing blockchain technology for P2P trading, and mainly solves the problems in conventional centralized or distributed P2P markets [19]–[20]. For example, to address the privacy and security problems, reference [20] combines blockchain with multi-signatures and anonymous message encryption. Nevertheless, the above studies apply blockchains for transactional data storage and money transfer, without integrating smart contracts. For automatic and fast execution of P2P energy trading, many studies refer to blockchain platforms that can support smart contracts, such as Ethereum [21], Hyperledger Fabric [22], and Hyperledger Besu [23]. However, the above platforms are either public blockchains with low performance or private blockchains with limited decentralization.

The second category focuses on the enhancement of existing blockchains without modifying their underlying structure, such as consensus protocol. For example, Reference [24] considers a blockchain where the consensus process is performed by a set of preselected aggregators, whereas [25] proposes a parallel double-chain structure with a voting-based consensus protocol. The authors in [26] propose a novel Proof of Stake consensus protocol to increase the self-consumption of renewable energy, and [27] adopts a reputation-based consensus protocol to quickly achieve consensus in the blockchain. However, the consensus protocols discussed are separated from the P2P trading mechanism, implying that the motivation of prosumers in validating transactions and managing the blockchain is neglected.

To overcome the aforesaid issues, a novel competition padding auction (CPA) mechanism is proposed for P2P energy trading among prosumers with private information on both reservation prices and quantity of energy to be traded. More specifically, the main contributions of the paper are:

1) A novel double auction mechanism for P2P energy trading among a community of prosumers is proposed, one which can promote local consumption of renewable energy-based generation and increase economic benefits of the prosumers.

2) The proposed CPA mechanism is rigorously proved to satisfy the properties of incentive compatibility, subsidy freedom, individual rationality and asymptotic economic efficiency. These properties demonstrate that the CPA mechanism overcomes the budget deficit of the VCG mechanism with a high level of economic efficiency, ensuring the sustainability and fairness of the P2P market.

3) The CPA-based P2P energy trading is implemented in a consortium blockchain, and a new consensus protocol that incorporates the CPA mechanism is presented, one which achieves high consensus efficiency and motivates prosumers to actively engage in managing the blockchain system.

The remainder of this paper is structured as follows. Section II presents the basic system model. The CPA mechanism and its implementation on blockchain are proposed in Section III, and Section IV gives the case studies in Portugal with 20 prosumers to show the effectiveness of the proposed mechanism. The conclusions are provided in Section V.

II. System Model

Figure 1 presents a structure of a smart community comprising multiple prosumers. Each prosumer is connected to the same distribution network, and equipped with PV systems and loads. Prosumers may or may not comprise BESS since deploying energy storage systems at the residential level can be costly.

In addition, all prosumers in a community are interconnected via the bidirectional power and information links, and the whole community is connected to the upstream utility grid through a single grid connection point. Every prosumer has a smart meter and an energy management system (EMS). The smart meter records the prosumer's generation, consumption, storage, and energy transaction with other prosumers and with the grid, and sends information to the EMS for processing.

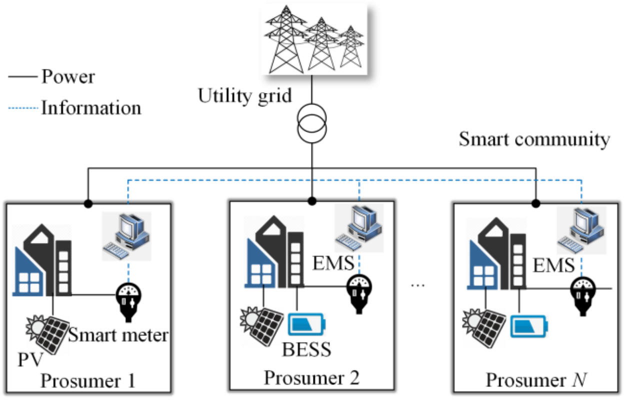


Fig. 1. Structure of a smart community for P2P trading.

A. Prosumer Models

With the existing infrastructures as explained, the main aim of this paper is to design a double auction mechanism for P2P energy trading in a private information environment. To this end, it is assumed that prosumers are capable of predicting the quantities of energy they generate and consume in a particular time period t. The quantities of energy generated and consumed by a prosumer $n \in \mathcal{N}$ at time period $t \in \mathcal{T} = \{1,2,\cdots,T\}$ are denoted by G_n^t and C_n^t, respectively. Thus, the generation-to-consumption ratio (GCR) of prosumer n at time period t is denoted by:

$$GCR_n^t = \frac{G_n^t}{C_n^t} \tag{1}$$

Let $\mathcal{N}_\text{d}^t = \{n \in \mathcal{N} | GCR_n^t \leqslant 1\}$ denote the set of demanders at time period t, indexed by i, with $N_\text{d}^t = |\mathcal{N}_\text{d}^t|$ for the total number of demanders. By the same token, $\mathcal{N}_\text{s}^t = \{n \in \mathcal{N} | GCR_n^t > 1\}$ denotes the set of suppliers at time period t, indexed by j, and $N_\text{s}^t = |\mathcal{N}_\text{s}^t|$ is the total number of suppliers. Clearly, $\mathcal{N}_\text{d}^t \cup \mathcal{N}_\text{s}^t = \mathcal{N}$ and $\mathcal{N}_\text{d}^t \cap \mathcal{N}_\text{s}^t = \varnothing$. Based on the value of *GCR*, the maximum amount of energy that demander $i \in \mathcal{N}_\text{d}^t$ can buy at period t is:

$$E_{i,\text{d}}^t = C_i^t (1 - GCR_i^t) \tag{2}$$

Similarly, the maximum amount of energy that supplier $j \in \mathcal{N}_\text{s}^t$ can sell at period t is:

$$E_{j,\text{s}}^t = C_j^t (GCR_j^t - 1) \tag{3}$$

As BESS are expensive, prosumers may or may not choose to use them. The presence of BESS introduces additional physical constraints. For each prosumer n that uses a BESS, $S_n^{t-\Delta t}$ and S_n^t denote the amount of energy stored in the system at the start and at the end of the time period t, respectively. Let $B_{n,\text{c}}^t$, $B_{n,\text{d}}^t$ denote the charging and discharging power; $\eta_{n,\text{c}}^s$, $\eta_{n,\text{d}}^s$ represent the charging and discharging efficiencies; and u_n^t, v_n^t denote the binary variables representing the charging and discharging states of the energy storage system. Assuming that the power levels remain constant during a given period, the dynamics of the energy level of the storage system can be modeled as [28]:

$$S_n^t = S_n^{t-\Delta t} + \left(u_n^t B_{n,\text{c}}^t \eta_{n,\text{c}}^s - \frac{v_n^t B_{n,\text{d}}^t}{\eta_{n,\text{c}}^s} \right) \Delta t, \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{4}$$

Note that S_n^t, $B_{n,\text{c}}^t$ and $B_{n,\text{d}}^t$ are all constrained within lower and upper limits. To avoid simultaneous charging and discharging of the BESS, the following constraint is satisfied, i.e.:

$$u_n^t + v_n^t \leqslant 1, \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{5}$$

In addition, the state of charge (SOC) of the BESS installed in prosumer n and in period t, is defined as:

$$SOC_n^t = \left(\frac{S_n^{t-\Delta t} - S_n^{\min}}{S_n^{\max} - S_n^{\min}} \right) \times 100\% \tag{6}$$

where $S_n^{\min}$ and $S_n^{\max}$ are the lower and upper bounds of the storage capacity of the BESS in prosumer n, respectively.

All prosumers have a connection to the utility grid for importing/exporting energy. Import and export prices are respectively denoted by λ^{im} and λ^{ex}, while to encourage self-consumption of PV generation, the export price is usually lower than the import price [29]. Although the power curves of PV systems have similar patterns because of the almost uniform distribution of solar radiation on prosumers in a community, the net power can vary widely across prosumers, who have different PV capabilities and loads. Therefore, P2P energy trading among prosumers all located in the same community is feasible.

B. Private Information of Prosumers

When participating in P2P trading at time period $t \in \mathcal{T}$, each supplier $j \in \mathcal{N}_\text{s}^t$ decides a reservation price μ_j^t. If the trading price, as received by supplier j per unit of energy, is lower than μ_j^t, then supplier j stops supplying energy in the P2P market. In the meantime, each demander $i \in \mathcal{N}_\text{d}^t$ determines a reservation price λ_i^t, i.e., the maximum price demander i is willing to pay in the P2P market for a unit of energy procured from suppliers. For trading to occur, it must be the case that reservation prices are bounded between λ^{ex} and λ^{im}. Indeed, there is no incentive for a prosumer to sell to peers at a lower price per unit of energy than is received by exporting to the grid, i.e., λ^{ex}. Moreover, should the reservation price of suppliers be higher than λ^{im}, prosumers would rather import energy from the grid. Similar arguments apply on the demand side of the P2P market.

As reservation prices depend on preferences and on the endowments of energy in each time period, the reservation price of each prosumer is modelled as a function of the SOC of the BESS. As the SOC changes from 0 to 100%, the mood of a supplier changes from being relaxed, moderate, and greedy, whereas the mood of a demander changes in the opposite direction, with a gradually decreasing price in both cases. The reservation price of demander $i \in \mathcal{N}_\mathrm{d}^t$ and supplier $j \in \mathcal{N}_\mathrm{s}^t$ are respectively modelled as:

$$\lambda_i^t = f_i^1(SOC_i^t) \text{ with } f_i^1(0) = \lambda^\mathrm{im}, f_i^1(100\%) = \lambda^\mathrm{ex} \quad (7)$$

$$\mu_j^t = g_j^1(SOC_j^t) \text{ with } g_j^1(0) = \lambda^\mathrm{im}, g_j^1(100\%) = \lambda^\mathrm{ex} \quad (8)$$

where both f_i^1 and g_i^1 are nonincreasing functions.

From (7)–(8), it can be observed that the reservation price of demander i and supplier j decreases as the SOC of the storage device increases, and is bounded between the import and export prices. In the case of $SOC_i^t = 0$ demander i's BESS cannot be discharged anymore, and the only outside option is to import energy from the grid, at the default price λ^im. As for supplier j with $SOC_j^t = 1$, an alternative to participating in P2P trading is to export to the grid at the price λ^ex. In addition, the prosumers without BESS have the reservation price λ^im when acting as demanders or the reservation price λ^ex as suppliers.

For each prosumer $n \in \mathcal{N}$, information related to the following variables is private:

1) G_n^t, PV generation;

2) C_n^t, energy consumption;

3) $E_{i,\mathrm{d}}^t$ or $E_{j,\mathrm{s}}^t$, maximum amount of energy demanded or supplied;

4) SOC_n^t, SOC of the BESS;

5) λ_i^t or μ_j^t, the per unit reservation price.

III. CPA Mechanism for P2P Energy Trading

In this section a double-auction-based P2P energy trading mechanism is designed to coordinate energy transactions among prosumers to reduce their dependence on the utility grid. As in other P2P market mechanisms proposed in [5] and [10], network constraints and transmission losses are not explicitly considered, and need to be addressed by a separate settlement process.

An efficient trading mechanism is a pivotal ingredient for the success of the P2P paradigm. In this paper, the application of double auction mechanisms is considered to facilitate P2P trading. This allows demanders and suppliers to balance their surplus energy and complementary demands by simultaneously presenting bids to buy and offers to sell. More specifically, the double-auction-based P2P trading within a community is conducted as follows. First, at each time period $t \in \mathcal{T}$, prosumers register into the P2P market as either a demander or a supplier, based on their generation and consumption. Second, each demander $i \in \mathcal{N}_\mathrm{d}^t$ submits a pair $(\lambda_i^{t'}, E_{i,\mathrm{d}}^{t'})$ that specifies its reservation price and the maximum quantity of demanded energy. Similarly, each supplier $j \in \mathcal{N}_\mathrm{s}^t$ submits a pair $(\mu_j^{t'}, E_{j,\mathrm{s}}^{t'})$ that specifies its reservation price and the maximum quantity of supplied energy. Third, based on the reservation prices in $\boldsymbol{\lambda}^{t'} = \{\lambda_i^{t'}\}_{i \in \mathcal{N}_\mathrm{d}^t}$ and $\boldsymbol{\mu}^t = \{\mu_j^{t'}\}_{j \in \mathcal{N}_\mathrm{s}^t}$, and on the maximum demanded and supplied energy quantities in $\boldsymbol{E}_\mathrm{d}^t = \{E_{i,\mathrm{d}}^{t'}\}_{i \in \mathcal{N}_\mathrm{d}^t}$ and $\boldsymbol{E}_\mathrm{s}^t = \{E_{j,\mathrm{s}}^{t'}\}_{j \in \mathcal{N}_\mathrm{s}^t}$, the P2P market is cleared according to a commonly known double auction mechanism. Notice that index t relates to the true private information of prosumers, while t' relates to the information submitted by prosumers.

The utility of prosumers as their net economic benefits from participating in P2P trading, on the demand and supply sides, are defined as:

$$\begin{aligned} u_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'}) = \lambda_i^t x_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'}) - \\ p_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'}), \forall i \in \mathcal{N}_\mathrm{d}^t, t \in \mathcal{T} \end{aligned} \quad (9)$$

$$\begin{aligned} v_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'}) = q_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'}) - \\ \mu_j^t y_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'}), \forall j \in \mathcal{N}_\mathrm{s}^t, t \in \mathcal{T} \end{aligned} \quad (10)$$

where $u_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'})$ and $v_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'})$ are the utility of demander i and supplier j; $x_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'})$ and $y_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'})$ are the amounts of energy bought by demander i and sold by supplier j; while $p_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'})$ and $q_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_\mathrm{d}^{t'}, \boldsymbol{E}_\mathrm{s}^{t'})$ are the payment by i and the revenue to j.

In (9), demand i's utility is defined as the cost savings from participating in P2P trading, as opposed to discharging the BESS and buying from the grid. Similarly, in (10), the utility to supplier j is defined as the revenue made in the P2P market, net of the revenue that would be received by charging the storages and exporting to the grid.

An efficient P2P market, by definition, maximizes the joint utility of all participating agents, and is not a priori biased in favour of demanders or suppliers. Ideally, economic efficiency should be obtained in a centralized way by letting prosumers submit their respective reservation prices as well as demand and supply quantities, before a price is computed at which socially optimal units of energy are bought or sold by each prosumer. However, this ideal approach is usually infeasible as a consequence of market participants reporting their private information strategically. For example, suppliers can significantly raise trading prices to obtain more benefits by underreporting their quantities [30] or by overreporting their reservation prices [31], resulting in a considerable loss of economic efficiency. Should P2P market operations be grounded on the information introduced by prosumers, a mechanism must be introduced to induce their truth-telling behaviour.

Definition 1: A double auction mechanism is weakly incentive compatible if:

1) Neither prosumer has an incentive to misreport its reservation price, i.e., given the truthful quantity information for any $i \in \mathcal{N}_{\mathrm{d}}^{t}$ and $j \in \mathcal{N}_{\mathrm{s}}^{t}$, it holds that:

$$u_i^t((\lambda_i^t, \boldsymbol{\lambda}_{-i}^{t'}), \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) \geqslant u_i^t((\lambda_i^{t'}, \boldsymbol{\lambda}_{-i}^{t'}), \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t), \quad \text{for any } \boldsymbol{\lambda}^{t'} \in R^{N_{\mathrm{d}}^t}, \boldsymbol{\mu}^{t'} \in R^{N_{\mathrm{s}}^t} \tag{11}$$

$$u_j^t(\boldsymbol{\lambda}^{t'}, (\mu_j^t, \boldsymbol{\mu}_{-j}^{t'}), \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) \geqslant u_j^t(\boldsymbol{\lambda}^{t'}, (\mu_j^{t'}, \boldsymbol{\mu}_{-j}^{t'}), \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t), \quad \text{for any } \boldsymbol{\lambda}^{t'} \in R^{N_{\mathrm{d}}^t}, \boldsymbol{\mu}^{t'} \in R^{N_{\mathrm{s}}^t} \tag{12}$$

where $\boldsymbol{\lambda}_{-i}^{t'} = \left\{ \lambda_k^{t'} \right\}_{k \in \mathcal{N}_d^t \setminus \{i\}}$; $\boldsymbol{\mu}_{-j}^{t'} = \left\{ \mu_l^{t'} \right\}_{l \in \mathcal{N}_s^t \setminus \{j\}}$; $\boldsymbol{E}_{-i,d}^{t'} = \left\{ E_{k,d}^{t'} \right\}_{k \in \mathcal{N}_d^t \setminus \{i\}}$; and $\boldsymbol{E}_{-j,s}^{t'} = \left\{ E_{l,s}^{t'} \right\}_{l \in \mathcal{N}_s^t \setminus \{j\}}$.

2) Neither prosumer has an incentive to misreport its maximum demand or supply quantity, given the truthful reservation prices, i.e., for any $i \in \mathcal{N}_{\mathrm{d}}^{t}$ and $j \in \mathcal{N}_{\mathrm{s}}^{t}$, it holds:

$$u_i^t(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, (E_{i,\mathrm{d}}^t, \boldsymbol{E}_{-i,\mathrm{d}}^{t'}), \boldsymbol{E}_{\mathrm{s}}^{t'}) \geqslant u_i^t(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, (E_{i,\mathrm{d}}^{t'}, \boldsymbol{E}_{-i,\mathrm{d}}^{t'}), \boldsymbol{E}_{\mathrm{s}}^{t'}), \quad \text{for any } \boldsymbol{E}_{\mathrm{d}}^{t'} \in R^{N_{\mathrm{d}}^t}, \boldsymbol{E}_{\mathrm{s}}^{t'} \in R^{N_{\mathrm{s}}^t} \tag{13}$$

$$u_j^t(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\mathrm{d}}^{t'}, (E_{j,\mathrm{s}}^t, \boldsymbol{E}_{-j,\mathrm{s}}^{t'})) \geqslant u_j^t(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\mathrm{d}}^{t'}, (E_{j,\mathrm{s}}^{t'}, \boldsymbol{E}_{-j,\mathrm{s}}^{t'})), \quad \text{for any } \boldsymbol{\lambda}^{t'} \in R^{N_{\mathrm{d}}^t}, \boldsymbol{\mu}^{t'} \in R^{N_{\mathrm{s}}^t} \tag{14}$$

When the weak incentive compatibility constraints in (11)–(14) are satisfied: there exists a dominant strategy equilibrium at which every prosumer reports its true reservation price, conditional on prosumers submitting quantity information truthfully; and there exists a dominant strategy equilibrium at which every prosumer reports its true quantity information, conditional on prosumers submitting their price information truthfully. A prosumer cannot thus profitably alter the trading price it faces by solely misreporting its reservation price or quantity information.

Definition 2: A double auction mechanism is subsidy-free if the total payments by demanders are weakly higher than the total rewards to suppliers, i.e.:

$$\sum_{i \in \mathcal{N}_{\mathrm{d}}^t} p_i^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}) \geqslant \sum_{j \in \mathcal{N}_{\mathrm{s}}^t} q_j^t(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'})$$

A subsidy-free mechanism allows for a non-negative budget surplus, which justifies the existence of the P2P market, otherwise the market needs to be subsidized by outside sources and would not be able to survive over the long term. In other words, the subsidy-free property ensures sustainability for a double-auction-based P2P market.

A. Competition Padding Auction Mechanism

The need for a subsidy of a VCG mechanism is a concern, because it does not guarantee the sustainability of the P2P approach for energy trading. To address the issue, a padding method is considered by introducing a phantom demander with a specific energy demand quantity denoted by Q^t and an infinite reservation price. The involvement of the phantom demander raises the equilibrium price, which is set as the buying price of demanders, and consequently, the total amount of energy traded may decrease. Given a smaller traded quantity, the selling price of suppliers is lower than the original equilibrium price. Therefore, the padding method can cover the budget deficit of the VCG mechanism by improving the buying price of demanders and reducing the selling price of suppliers. Inspired by the padding method, a demander competition padding auction (D-CPA) is designed for P2P energy trading by adopting the padding method. It includes the following three steps.

In Step 1 (padding setting), the padding term is set as $Q^t = Q(\boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}) : R_+^N \to R_+$, which is a nonnegative function of the submitted demand and supply quantities.

In Step 2 (primary market clearing), considering the padding term Q^t, the following primary market clearing problem $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t, \mathcal{N}_{\mathrm{s}}^t)$ is solved:

$$H(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}) = \max_{(\boldsymbol{x}^t, \boldsymbol{y}^t)} \sum_{i \in \mathcal{N}_{\mathrm{d}}^t} \lambda_i^{t'} x_i^t - \sum_{j \in \mathcal{N}_{\mathrm{s}}^t} \mu_j^{t'} y_j^t, \quad \text{s.t.} \sum_{i \in \mathcal{N}_{\mathrm{d}}^t} x_i^t + Q^t = \sum_{j \in \mathcal{N}_{\mathrm{s}}^t} y_j^t \tag{15a}$$

$$0 \leqslant x_i^t \leqslant E_{i,\mathrm{d}}^{t'}, \forall i \in \mathcal{N}_{\mathrm{d}}^t \tag{15b}$$

$$0 \leqslant y_j^t \leqslant E_{j,\mathrm{s}}^{t'}, \forall j \in \mathcal{N}_{\mathrm{s}}^t \tag{15c}$$

We denote $(\boldsymbol{x}^{t*}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}), \boldsymbol{y}^{t*}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}))$ as the solution of $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t, \mathcal{N}_{\mathrm{s}}^t)$. The set of remaining demanders, who in the solution are allocated all of their submitted demanded quantities, is denoted as $\mathcal{N}_{\mathrm{d}}^t \triangleq \left\{ i \in \mathcal{N}_{\mathrm{d}}^t | x_i^{t*}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}) = E_{i,\mathrm{d}}^{t'} \right\}$. The demanders in $\mathcal{N}_{\mathrm{d}}^t \setminus \mathcal{N}_{\mathrm{d}}^t$ are excluded from the final allocation. We denote $\alpha_i^t(\boldsymbol{\lambda}_{-i}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'})$ the infimum of reservation prices resulting in the demander $i \in \mathcal{N}_{\mathrm{d}}^t$ to be allocated all of its submitted demand in the solution to the problem $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t, \mathcal{N}_{\mathrm{s}}^t)$. Its mathematical description is:

$$\alpha_i^t(\boldsymbol{\lambda}_{-i}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}) = \inf \left\{ \lambda_i^{t'} | x_i^{t*}((\lambda_i^{t'}, \boldsymbol{\lambda}_{-i}^{t'}), \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^{t'}, \boldsymbol{E}_{\mathrm{s}}^{t'}) = E_{i,\mathrm{d}}^{t'} \right\} \tag{16}$$

In Step 3 (secondary market clearing), given the remaining demanders and their submitted reservation prices, the following secondary market clearing problem $\mathcal{S}(\mathcal{N}_{\mathrm{d}}^t, \mathcal{N}_{\mathrm{s}}^t)$ is solved:

$$M(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})=\max_{(\boldsymbol{x}^t,\boldsymbol{y}^t)}\sum_{i\in\mathcal{N}_{\rm d}^t}\lambda_i^{t'}x_i^t-\sum_{j\in\mathcal{N}_{\rm s}^t}\mu_j^{t'}y_j^t$$
$$\text{s.t.}\sum_{i\in\mathcal{N}_{\rm d}^t}x_i^t=\sum_{j\in\mathcal{N}_{\rm s}^t}y_j^t \tag{17a}$$
$$0\leqslant x_i^t\leqslant E_{i,{\rm d}}^{t'},\forall i\in\mathcal{N}_{\rm d}^t \tag{17b}$$
$$0\leqslant y_j^t\leqslant E_{j,{\rm s}}^{t'},\forall j\in\mathcal{N}_{\rm s}^t \tag{17c}$$

Denoting $(\boldsymbol{x}^{t**}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t),\boldsymbol{y}^{t**}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t))$ as the solution to $\mathcal{S}(\mathcal{N}_{\rm d}^t,\mathcal{N}_{\rm s}^t)$, the energy allocation rule of D-CPA is:

$$x_i^{t({\rm D-CPA})}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})=\begin{cases}x_i^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'}), \text{if } i\in\mathcal{N}_{\rm d}^t\\ 0, \quad \text{if } i\in\mathcal{N}_{\rm d}^t\setminus\mathcal{N}_{\rm d}^t\end{cases} \tag{18}$$

$$y_j^{t({\rm D-CPA})}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})=y_j^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'}),\forall j\in\mathcal{N}_{\rm s}^t \tag{19}$$

In contrast to the VCG mechanism, the payment rule of the D-CPA varies between demanders and suppliers, and is defined by:

$$p_i^{t({\rm D-CPA})}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})=\begin{cases}E_{i,{\rm d}}^{t'}\alpha_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'}), \text{if } i\in\mathcal{N}_{\rm d}^t\\ 0, \quad \text{if } i\in\mathcal{N}_{\rm d}^t\setminus\mathcal{N}_{\rm d}^t\end{cases} \tag{20}$$

$$q_j^{t({\rm D-CPA})}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})=\mu_j^{t'}y_j^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})+(M(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{\rm s}^{t'})-M_{-j}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-j}^{t'},\boldsymbol{E}_{\rm d}^{t'},\boldsymbol{E}_{-j,{\rm s}}^{t'})),\forall j\in\mathcal{N}_{\rm s}^t \tag{21}$$

where $M_{-j}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-j}^{t'},\boldsymbol{E}_{\rm s}^{t'},\boldsymbol{E}_{-j,{\rm s}}^{t'})$ represents the maximum welfare of the remaining demanders and suppliers without supplier *j*. In the following, the weak incentive compatibility of D-CPA is established, before characterizing its performance in terms of budget and economic efficiency.

The total amount of energy provided by all suppliers in time period $t\in\mathcal{T}$ is $\sum_{j\in\mathcal{N}_{\rm s}^t}E_{j,{\rm s}}^{t'}$. The supplied quantities in relation to the submitted reservation prices are ranked first, and the inverse form of the supply function is then denoted by $s^t(h)$, i.e., the lowest offer that can supply h units of energy in time period t. Similarly, the inverse demand function is defined as $d^t(h)$. Based on the functions of $s^t(h)$ and $d^t(h)$, Fig. 2 illustrates the solution to $\mathcal{P}(\mathcal{N}_{\rm d}^t,\mathcal{N}_{\rm s}^t)$. Finally, the inverse supply function is denoted by $s_{-j}^t(h)$ when supplier j is not present, i.e., the lowest offer among suppliers except supplier j that can supply h units of energy. Clearly, both $s^t(h)$ and $s_{-j}^t(h)$ are nondecreasing in h, and for any $j\in\mathcal{N}_{\rm s}^t$, it holds that:

$$s^t(h+E_{j,{\rm s}}^{t'})\geqslant s_{-j}^t(h),\forall h\geqslant 0 \tag{22}$$

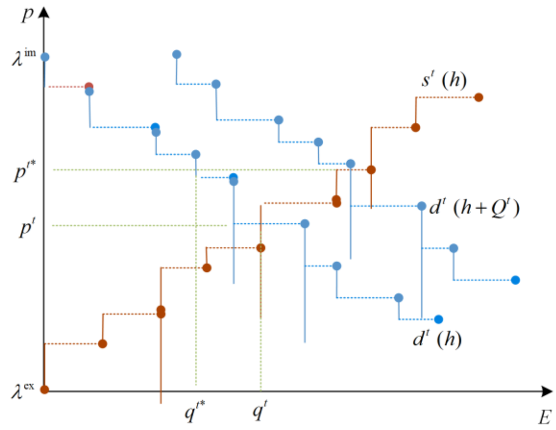


Fig. 2. A graphical representation of solution to $\mathcal{P}(\mathcal{N}_{\rm d}^t,\mathcal{N}_{\rm s}^t)$.

Proposition 1: Every demander $i\in\mathcal{N}_{\rm d}^t$ faces a unit buying price $\alpha_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t)$ and it is a dominant strategy to submit its true reservation price, given the truthful quantity information of all prosumers.

The proofs of all propositions and lemmas are given in the Appendix A.

Proposition 1 shows that each demander is incentivized to submit its true reservation price, regardless of the bidding behaviour of other prosumers. As for the dominant strategy equilibrium, the remaining demander i actually faces a unit purchasing price of $\alpha_i^t(\boldsymbol{\lambda}_{-i}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t)$. Although calculating α_i^t based on its definition given by (16) can be technically challenging, the economic implication of α_i^t inspires an appropriate approach.

Lemma 1: The unit buying price of any remaining demander i is:

$$\alpha_i^t(\boldsymbol{\lambda}_{-i}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t)=\lambda_i^t-\gamma_i^t(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t) \tag{23}$$

where $\gamma_i^t(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\rm d}^t,\boldsymbol{E}_{\rm s}^t)>0$ is the shadow price of the demanded quantity constraint $x_i^t\leqslant E_{i,{\rm d}}^t$. All remaining demanders share the same buying price.

Lemma 1 establishes that when the shadow price associated with a demander's maximum demanded quantity constraint is positive, i.e., the welfare change induced by an increase in the demanded quantity is positive, the needed quantity can be procured entirely from the P2P market. In this case, the unit utility γ_i^t is equal to the positive shadow price. Moreover, all remaining demanders face a uniform unit purchasing price. This interpretation helps computing the latter purchasing price.

Proposition 2: Given the truthful quantity information of all prosumers, it is a dominant strategy for any supplier $j\in\mathcal{N}_{\rm s}^t$ to submit its true reservation price if

$Q^t = Q(\boldsymbol{E}_{\rm d}^t, \boldsymbol{E}_{\rm s}^t) \geqslant \max\{E_{j,{\rm s}}^t \mid j \in \mathcal{N}_{\rm s}^t\}$.

Generally speaking, when many suppliers sell their excess energy in the P2P market, the one with the largest supply, $\max\{E_{j,{\rm s}}^t \mid j \in \mathcal{N}_{\rm s}^t\}$, may have the largest ability to manipulate the trading price. Proposition 2 shows that a padding term higher than $\max\{E_{j,{\rm s}}^t \mid j \in \mathcal{N}_{\rm s}^t\}$ is sufficient to counteract such a market power and to guarantee a truthful bidding behaviour of suppliers. Moreover, unlike in the case of demanders, each supplier faces a nonuniform price scheme for selling energy. In this paper, it now assumes that suppliers will not overstate the quantity they supply because of the risk of being "punished" in case the P2P contract cannot be executed. Based on this assumption, it can be seen that incentive compatibility relative quantity information presumes that prosumers submit their true reservation price.

Proposition 3: Any demander $i \in \mathcal{N}_{\rm d}^t$ submits the true demand quantity in the following two cases: 1) $N_{\rm d}^t + N_{\rm s}^t = \infty$; and 2) $N_{\rm d}^t + N_{\rm s}^t < \infty$ and $F(x) \leqslant \frac{x}{R^t - x}, x \in \left[0, \frac{R^t}{2}\right]$, where $F(x)$ is the distribution of the allocation quantity of the marginal prosumer, and $R^t = \max\{E_{i,{\rm d}}^t \mid i \in \mathcal{N}_{\rm d}^t\}$. Any supplier $j \in \mathcal{N}_{\rm s}^t$ always submits its true supply quantity.

Without prior knowledge of the price and quantity information of prosumers, demanders may assume that the allocation of the marginal prosumer follows a uniform distribution, i.e., $F(x) = \frac{x}{R^t}$, $x \in \left[0, R^t\right]$. In this case, the sufficient condition on $F(x)$ is satisfied and demanders have incentives to submit their true quantity. Based on the truth-telling equilibrium, it can be easily observed that each prosumer obtains a non-negative utility from participating in the P2P energy market. Therefore, the D-CPA mechanism satisfies individual rationality. Proposition 4 below gives a sufficient condition for the proposed mechanism to be subsidy-free.

Proposition 4: The D-CPA mechanism is subsidy-free:

$$\sum_{i \in \mathcal{N}_{\rm d}^t} p_i^{t({\rm D-CPA})}(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\rm d}^t, \boldsymbol{E}_{\rm s}^t) \geqslant \sum_{j \in \mathcal{N}_{\rm s}^t} q_j^{t({\rm D-CPA})}(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\rm d}^t, \boldsymbol{E}_{\rm s}^t),$$
$$\text{when } Q^t \geqslant \max\left\{E_{j,{\rm s}}^t | j \in \mathcal{N}_{\rm s}^t\right\} \tag{24}$$

Proposition 4 establishes that the D-CPA mechanism satisfies the subsidy-free property when the padding term is sufficiently high, i.e., weakly greater than $\max\left\{E_{j,{\rm s}}^t | j \in \mathcal{N}_{\rm s}^t\right\}$, a condition that also implies incentive compatibility. To illustrate, the following examples are considered where a budget deficit results from the above condition not being satisfied. Suppose that only one supplier and one demander participate in the P2P market. The supplier sells $E_{\rm s}$ kWh at the reservation price p €/kWh. With the padding term $Q^t < E_{\rm s}$, the demanded energy is $E_d = (E_{\rm s} - Q^t)$kWh and the reservation price is q €/kWh, where $q > p$. From the allocation and payment rules defined by (18)–(21), the demander buys $E_{\rm d}$ kWh and pays €$pE_{\rm d}$, while the supplier sells $E_{\rm d}$ kWh and earns €$qE_{\rm d}$. Thus, the D-CPA mechanism may result in a budget deficit when $Q^t < \max\left\{E_{j,{\rm s}}^t | j \in \mathcal{N}_{\rm s}^t\right\}$. Note that the overall social welfare reduces as the padding term increases. To achieve a high level of economic efficiency and ensure subsidy freedom, the padding term can be set as $Q^t = \max\left\{E_{j,{\rm s}}^t | j \in \mathcal{N}_{\rm s}^t\right\}$ in the D-CPA mechanism.

Note that the D-CPA mechanism allocates goods in a way that maximizes the joint utility of the remaining demanders and all suppliers. In other words, the welfare of all market participants cannot be maximized and the D-CPA may result in a welfare loss. However, it is established that the proposed auction achieves economic efficiency asymptotically, implying that the welfare loss converges to zero as the number of prosumers approaches infinity.

Proposition 5: The D-CPA mechanism is asymptotically efficient:

$$\lim_{N_{\rm d}^t + N_{\rm s}^t \to \infty} \frac{G(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\rm d}^t, \boldsymbol{E}_{\rm s}^t) - M(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\rm d}^t, \boldsymbol{E}_{\rm s}^t)}{G(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t, \boldsymbol{E}_{\rm d}^t, \boldsymbol{E}_{\rm s}^t)} = 0 \tag{25}$$

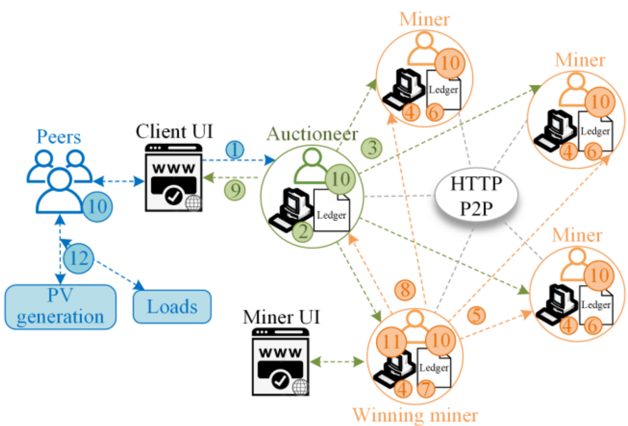
Therefore, the D-CPA mechanism outperforms the VCG mechanism in a sufficiently large P2P market since the former is efficient in the limit while satisfying the subsidy-free property.

In the P2P market, demanders and suppliers are supposed to be treated equally and the auction mechanism should not be biased in favour of either side. While Section III-A designs a D-CPA mechanism by introducing a padding term on the demand side, the S-CPA can be similarly structured by considering a padding term on the supply side. Computing and comparing welfare across the two cases, the CPA mechanism is defined as the alternative that yields the higher welfare level. Therefore, the CPA mechanism is also weakly incentive compatible and subsidy-free when the relevant sufficient conditions given in Propositions 3 and 4 are satisfied. Similar to Proposition 5, it can verify the asymptotical economic efficiency of the CPA mechanism.

B. Implementation of CPA Mechanism on Blockchain

By adopting blockchain technologies, the CPA mechanism can be implemented in a secure, verifiable and automated manner. In this paper, a consortium blockchain infrastructure is exploited to offer permissioned ledgers and trading managements. Compared with public blockchains, consortium blockchains are more suitable for the decentralized P2P trading as it is necessary to verify the identity of the prosumers. In addition, consortium blockchains are more decentralized, secure, and trustworthy than private blockchains.

The distribution system operator verifies the identity of every prosumer and further classifies all prosumers or peers from a system viewpoint in the smart community into normal peers and validator peers according to the computational capabilities. In addition, the distribution system operator periodically distributes an identification number to each peer to participate in P2P trading. Normal peers participate in P2P energy trading and hold a copy of ledgers. In addition to taking part in transactions, validator peers are responsible for approving modifications of the ledger and for reaching consensus regarding the valid state of the ledger. In a localized community with blockchain-based P2P energy trading, since there exists no central authority to manage transactions, all peers must agree on a transaction or a block (a group of transactions) before storing it in the ledger. The validity of a new transaction holds if and only if consensus is reached among all validator peers. To expedite the computing process, an improved consensus protocol is proposed, which decomposes and interacts with the CPA mechanism-based auction process, as described in Fig. 3.



1. Peers submit buy or sell orders via the client UI.
2. The auctioneer organizes data.
3. The auctioneer broadcasts data to miners.
4. Miners compete to solve the clearing problems $\mathcal{P}(\mathcal{N}_d^t,\mathcal{N}_s^t)$ and $(\mathcal{N}_d^t,\mathcal{N}_s^t)$ via the miner UI.
5. The miner who first solves the problems wins and broadcasts its solutions.
6. All miners validate the trading results.
7. The winning miner processes the validated results and hashes the new block with transactions.
8. The winning miner broadcasts the new block to the auctioneer and all miners.
9. Auctioneer broadcasts trading results and block information to all peers.
10. All peers update ledgers to achieve consensus.
11. The winner miner gets reward, which is the budget surplus of the CPA mechanism.
12. Smart contracts are executed.

Fig. 3. The improved consensus protocols for consortium blockchain.

IV. Case Studies

In this section, the effectiveness of the proposed blockchain-based CPA mechanism for P2P energy trading is validated with multiple experiments, both within a simulation environment and on a hardware system.

A. Basic Setting

A smart community in Portugal with 20 prosumers is studied. Each prosumer has baseloads and PV systems, while prosumers 5–20 own BESS. The daily load profiles and PV generation curves of all prosumers, and the operating parameters of BESS are taken from [32]. The export price in Portugal is $\lambda^{ex} = 0.041$ €/kWh [33]. For one day the case study considers $\mathcal{T} = \{1,\cdots,24\}$. Without loss of generality, a time period $t = 9$ is selected to conduct a simulation analysis for P2P trading because in this period demanders and suppliers simultaneously exist. The import price at $t = 9$ is $\lambda^{im} = 0.13$ €/kWh [31]. First, for Prosumers 1–4 without BESS, their reservation price is $\lambda^{im} = 0.13$ €/kWh when acting as a demander, while it is $\lambda^{ex} = 0.041$ €/kWh when acting as a supplier. Second, for Prosumers 5–20, who are equipped with BESS, it assumes that their reservation price, which is bounded between λ^{ex} and λ^{im}, is a linear function of the SOC of the BESS. The value of the SOC is generated via a uniform distribution, and the reservation price of each prosumer is calculated according to the above assumptions, as presented in Table I.

TABLE I
Private Information of All Prosumers

Prosumer ID	Reservation price (€/kWh)	PV generation (kWh)	Baseload (kWh)	Demand quantity (kWh)	Supply quantity (kWh)
1	0.13	59.68	64.78	5.1	
2	0.13	59.68	69.38	9.7	
3	0.041	59.68	51.58		8.1
4	0.13	59.68	61.13	1.45	
5	0.057	59.68	51.33		8.35
6	0.0686	59.68	71.02	11.3	
7	0.1015	59.68	54.18		5.5
8	0.0455	59.68	68.18	8.5	
9	0.1273	59.68	54.1		5.58
10	0.0908	59.68	67.54	7.86	
11	0.0742	59.68	53.84		5.84
12	0.0633	59.68	70.3	10.6	
13	0.0953	59.68	56.4		3.28
14	0.0713	59.68	52.72		6.96
15	0.1149	59.68	54.03		5.65
16	0.0677	59.68	62.79	3.11	
17	0.1273	59.68	59.36		0.32
18	0.1051	59.68	56.44		3.24
19	0.1264	59.68	67.94	8.26	
20	0.1211	59.68	62.05	2.37	

A blockchain system is set up in the laboratory partly inspired by [34]. The validator network is formed by 10 prosumers running on their personal computers. The user interfaces for all prosumers to input their buy or sell orders. The validators implement the proposed consensus protocol in Section III-B by Python 3.8. A pair of public and private keys are generated for each peer via the Round Sheep Hash cryptography toolbox. For traceability and verification, the hash function SHA-256 is employed to generate cryptographic hash values for blocks in the consortium blockchain. When a validator peer receives all private data from an auctioneer peer, it solves the clearing problem by using the SciPY optimizer. For reference, each personal computer carries a 2.0 GHz Core i7-10700T CPU and 16 GB RAM. The P2P market is cleared separately for each time period.

B. Equilibrium Outcome Analysis

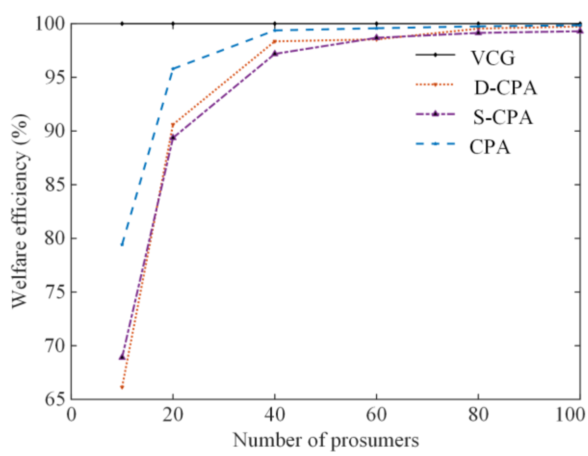
To evaluate the performance of the proposed double auction mechanism, the P2P energy trading outcomes are compared under two double auctions: the VCG and the CPA mechanisms. To ensure truth-telling equilibrium, the padding term is set as $\max\left\{E^{9}_{j,\mathrm{s}} | j \in \mathcal{N}^{9}_{\mathrm{s}}\right\} = 8.35$ kWh in the D-CPA mechanism, and $\max\left\{E^{9}_{i,\mathrm{d}} | i \in \mathcal{N}^{9}_{\mathrm{d}}\right\} =$ 11.3 kWh in the S-CPA mechanism. Table II shows the equilibrium of the three mechanisms and the following results can be observed. First, the VCG mechanism maximizes total welfare by not excluding any profitable energy transactions. Second, the D-CPA mechanism excludes a demander (Prosumer 10) by introducing a phantom demander with demanded quantity 8.35 kWh and unlimited budget, resulting in a welfare reduction. Third, the S-CPA mechanism eliminates a supplier (Prosumer 11) by adding a phantom supplier with a supplied quantity of 11.3 kWh and a reservation price equal to zero. Accordingly, the profitable transactions of Prosumer 11 with Prosumers 10 and 20 are cancelled. Finally, both CPA mechanisms result in less social welfare than in the VCG scenario, while they ensure a budget surplus for the sustainability of P2P trading.

The equilibrium welfare and budget performance of the three weakly incentive compatible mechanisms are compared for chosen numbers of prosumers, namely N equals to 20, 40, 60, 80, and 100, which represent different sizes of the trading community. For the experiments to be reliable, the average equilibrium outcomes over 100 instances for each size of the trading community are obtained, with randomly generated private information. The maximum social welfare produced by the VCG mechanism is considered as a benchmark, while welfare efficiency is defined as the percentage of a given mechanism's welfare level relative to the one resulting from the VCG mechanism. Welfare efficiencies with the four types of double auction mechanisms, for each size of the trading community, are represented in Fig. 4(a). As seen, the CPA mechanism improves the welfare performance of the D-CPA and S-CPA mechanisms. Either the D-CPA or the S-CPA mechanism may reach a higher welfare level depending on the private information of prosumers. In addition, all three types of CPA mechanisms achieve asymptotic economic efficiency, as their welfare efficiency measures exceed 99% when the number of prosumers is 100. The budget performances of the four mechanisms shown in Fig. 4(b) indicate that the CPA mechanisms scenarios always generate a budget surplus, while the VCG mechanism always runs with a deficit. Figure 4 also shows some decision supports for the community to choose a sustainable subsidy-free double auction mechanism. When the community size is not large enough, e.g., $N \leqslant 60$, the CPA mechanism achieves the highest welfare level with a positive budget surplus. However, for a large community, e.g., $N \geqslant 80$, it is suggested to implement either the D-CPA mechanism or the S-CPA mechanism, since they encourage the validator peers to manage and validate large numbers of transactions by setting higher rewards without much welfare loss.

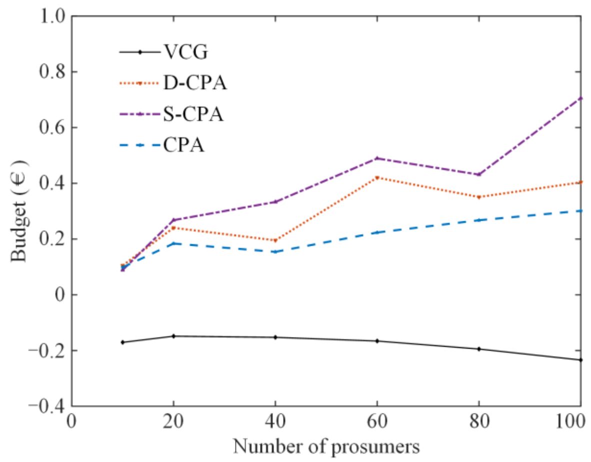
TABLE II
EQUILIBRIUM OF THREE AUCTION MECHANISMS

Prosumer ID	VCG		D-CPA		S-CPA	
	Allocation (kWh)	Payment (€)	Allocation (kWh)	Payment (€)	Allocation (kWh)	Payment (€)
1	+5.1	0.4631	+5.1	0.5177	+5.1	0.5741
2	+9.7	0.8109	+9.7	0.9846	+9.7	0.9917
3	−8.1	−0.7765	−8.1	−0.7371	−8.1	−0.601
4	+1.45	0.1317	+1.45	0.1472	+1.45	0.1814
5	−8.35	−0.8018	−8.35	−0.7625	−8.35	−0.6196
10	+2.37	0.1759	0	0	0	0
11	−5.84	−0.5471	−3.47	-0.3319	0	0
14	−6.96	−0.6607	−6.96	-0.6214	−6.96	−0.5164
19	+8.26	0.704	+8.26	0.8384	+7.16	0.7219
20	+2.37	0.2152	+2.37	0.2406		
Social welfare	1.9211		1.8818		1.7132	
Profit	−0.2853		0.2756		0.7321	

Note: "+/−" in the payment column denotes buy/sell; "−" in the payment column denotes revenues.



(a)



(b)

Fig. 4. Comparison of equilibrium outcome of different double auction mechanisms. (a) Welfare efficiency of VCG, D-CPA, S-CPA, and CPA. (b)Budget efficiency of VCG, D-CPA, S-CPA, and CPA.

C. Performance of the Blockchain System

Figure 5 illustrates the evaluation of the scalability and safety of P2P trading. Good scalability implies that the blockchain-enabled trading system performs well as it grows in scale. The percentage of miners stands for the decentralization level of the blockchain system. With more miners validating transactions and managing ledgers, the decentralization and safety of the blockchain-enabled trading system are improved. When the number of prosumers increases from 20 to 100 and the percentage of miners increases from 25% to 100%, as shown in Fig. 5, the time of mining or clearing the P2P market is limited within seconds. Therefore, our consortium blockchain system satisfies the speed requirement of real-time energy transactions, which are usually conducted every fifteen minutes. In addition, both increased number of prosumers and raised percentage of miners require more time for achieving consensus, with the former having a stronger effect. Therefore, the scalability requires more attention when designing a blockchain-enabled trading system.

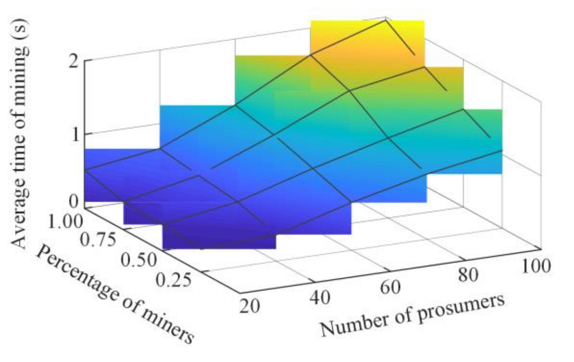


Fig. 5. The time of generating a new block with different numbers of prosumers and miners.

V. Conclusion

A blockchain-enable decentralized P2P market is investigated where prosumers have private information about the amount of energy to be traded and reservation prices. A novel CPA mechanism is designed, one which outperforms the VCG mechanism since it achieves asymptotic economic efficiency and guarantees sustainability by not resulting in any budget deficit. To offer a high level of security and automation for P2P energy transactions, the proposed mechanism is implemented by blockchain smart contracts. An improved consensus protocol is designed by incorporating the proposed mechanism. This provides a fast consensus process and meets the speed and scalability requirements for real time P2P trading, while the budget surplus of the mechanism motivates prosumers to manage the blockchain.

The following considerations will be made in future research. First, this paper only considers weak incentive compatibility in auction design, i.e., the possibility that prosumers exercise their market power by simultaneously misreporting their quantity and price information is not included. Second, because prosumers only trade a small amount of energy, it assumes sufficient capacities along the power lines connecting demanders and suppliers. However, when larger prosumers, such as industrial prosumers and local business prosumers, participate in P2P trading, it is necessary to consider the transmission constraints and investigate how prosumers modify their bidding strategies depending on transmission capacities under different auction mechanisms.

Appendix A

Proof of Proposition 1: For any demander $i \in \mathcal{N}_{\mathrm{d}}^t$, if the submitted price $\lambda_i^{t'}$ verifies $\lambda_i^{t'} < \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t)$, then $x_i^{t*}((\lambda_i^{t'}, \boldsymbol{\lambda}_{-i}^{t'}), \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) < E_{i,\mathrm{d}}^t$ and $i \in \mathcal{N}_{\mathrm{d}}^t \setminus \mathcal{N}_{\mathrm{d}}^t$. In this case, the utility of demander i is zero. If $\lambda_i^{t'} > \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t)$, it then immediately follows that $x_i^{t*}((\lambda_i^{t'}, \boldsymbol{\lambda}_{-i}^{t'}), \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) = E_{i,\mathrm{d}}^t$. So there is $\lambda_i^{t'} \geqslant s^t\left(\sum_{i \in \mathcal{N}_{\mathrm{d}}^t} x_i^{t*}((\lambda_i^{t'}, \boldsymbol{\lambda}_{-i}^{t'}), \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) + Q^t\right)$, and $x_i^{t**}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) = E_{i,\mathrm{d}}^t$. If $x_i^{t**}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) < E_{i,\mathrm{d}}^t$, it can find a constant $\varepsilon > 0$ such as (A.1) which is a contradiction：

$$\begin{aligned}
\lambda_i^{t'} < s^t\left(\sum_{i \in \mathcal{N}_{\mathrm{d}}^t} x_i^{t**}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) + \varepsilon\right) \leqslant \\
s^t\left(\sum_{i \in \mathcal{N}_{\mathrm{d}}^t} E_{i,d}^t + Q^t\right) \leqslant \qquad \text{(A.1)} \\
s^t\left(\sum_{i \in \mathcal{N}_{\mathrm{d}}^t} x_i^{t*}(\boldsymbol{\lambda}^{t'}, \boldsymbol{\mu}^{t'}, \boldsymbol{E}_{\mathrm{d}}^t, \boldsymbol{E}_{\mathrm{s}}^t) + \varepsilon\right) \leqslant \lambda_i^{t'}
\end{aligned}$$

Thus, there is $x_i^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=E_{i,\mathrm{d}}^t$ for $\lambda_i^{t'} > \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$.

To conclude, if the true reservation price λ_i^t of demander i satisfies $\lambda_i^t > \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$, then it prefers buying all of its demand from the P2P market since each unit generates a positive utility, which can be achieved by submitting λ_i^t. If $\lambda_i^t < \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$, then the best outcome for demander i is not to buy anything and this can be achieved by submitting λ_i^t. If $\lambda_i^t = \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$, then demander i either trades for a utility equal to zero or does not trade. In this case, submitting λ_i^t is also the best response. Therefore, demander i faces a take-it-or-leave-it offer at unit price $\underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$ and it is a dominant strategy to submit her true reservation price.

Proof of Lemma 1: First, the following equivalence between three conditions can be observed:

$$\begin{aligned}
& x_i^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=E_{i,\mathrm{d}}^t \Leftrightarrow \\
& \quad \Delta H(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},(E_{i,\mathrm{d}}^t+\varepsilon,\boldsymbol{E}_{-i,\mathrm{d}}^t),\boldsymbol{E}_{\mathrm{s}}^t) > \\
& \quad H(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t), \text{ for any } \varepsilon>0 \Leftrightarrow \\
& \quad \gamma_i^t(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)>0
\end{aligned} \tag{A.2}$$

Second, it holds that $\lambda_i^t > \underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$ if and only if $\gamma_i^t((\lambda_i^t,\boldsymbol{\lambda}_{-i}^{t'}),\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)>0$.

Third, in the following it is proved that $\underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=\lambda_i^t-\gamma_i^t((\lambda_i^t,\boldsymbol{\lambda}_{-i}^{t'}),\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$ for any demander $i\in\mathcal{N}_{\mathrm{d}}^t$. This is achieved by showing that when λ_i^t increases from $\underline{p}_i^t$ to $\underline{p}_i^t+\Delta$, γ_i^t will increase from 0 to Δ for any $\Delta>0$. It follows from the above that $\gamma_i^t=0$ when $\lambda_i^t=\underline{p}_i^t$. When λ_i^t increases from $\underline{p}_i^t$ to $\underline{p}_i^t+\Delta$, $H((\lambda_i^t,\boldsymbol{\lambda}_{-i}^{t'}),\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$ increases by $\Delta E_{i,\mathrm{d}}^t, H((\lambda_i^t,\boldsymbol{\lambda}_{-i}^{t'}),\boldsymbol{\mu}^{t'},(E_{i,\mathrm{d}}^t+\varepsilon,\boldsymbol{E}_{-i,\mathrm{d}}^t),\boldsymbol{E}_{\mathrm{s}}^t)$ increases by $\Delta(E_{i,\mathrm{d}}^t+\varepsilon)$. Thus, γ_i^t increases by $\dfrac{\Delta(E_{i,\mathrm{d}}^t+\varepsilon)-\Delta E_{i,\mathrm{d}}^t}{\varepsilon}=\Delta.$ Therefore, $\underline{p}_i^t=\lambda_i^t-\gamma_i^t$.

Finally, it will be shown that for any two demanders $i,k\in\mathcal{N}_{\mathrm{d}}^t$, $\underline{p}_i^t(\boldsymbol{\lambda}_{-i}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=\underline{p}_k^t(\boldsymbol{\lambda}_{-k}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$ holds. Let λ and ρ_i denote the Lagrange multipliers associated with the constraints (15a) and (15b). From the Karush-Kuhn-Tucker conditions, there is $\lambda_i^t-\lambda-\rho_i=0$ for demander $i\in\mathcal{N}_{\mathrm{d}}^t$. Since the Lagrange multiplier ρ_i measures the change in the objective function implied by a change in $E_{i,\mathrm{d}}^t$, there is $\rho_i=\gamma_i^t$. The above results lead to $\underline{p}_i^t=\lambda_i^t-\gamma_i^t=\lambda_i^t-\rho_i=\lambda_k^t-\rho_k=\lambda_k^t-\gamma_k^t=\underline{p}_k^t$.

Proof of Proposition 2: To prove proposition 2, the following three lemmas are demonstrated based on the assumption: $Q^t \geqslant \max\left\{E_{j,\mathrm{s}}^t \mid j\in\mathcal{N}_{\mathrm{s}}^t\right\}$.

Lemma A.1: For any supplier $l\in\mathcal{N}_{\mathrm{s}}^t$, there is $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=0$ if $y_l^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)<E_{l,\mathrm{s}}^t$.

Proof of Lemma A.1: First, the set of suppliers with a positive allocation in the problem $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t,\mathcal{N}_{\mathrm{s}}^t)$ is denoted by $\mathcal{N}_s^{\prime t}=\left\{j\in\mathcal{N}_s^t \mid y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)>0\right\}$. Then an upper bound of the total cleared quantities in the problem $\mathcal{S}(\mathcal{N}_{\mathrm{d}}^t,\mathcal{N}_{\mathrm{s}}^t)$ can be obtained as:

$$\begin{aligned}
& \sum_{i\in\mathcal{N}_{\mathrm{d}}^t} x_i^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=\sum_{i\in\mathcal{N}_{\mathrm{d}}^t}E_{i,\mathrm{d}}^t \leqslant \\
& \sum_{i\in\mathcal{N}_{\mathrm{d}}^t} x_i^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)= \\
& \sum_{j\in\mathcal{N}_{\mathrm{s}}^t} y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)-Q^t= \\
& \sum_{j\in\mathcal{N}_{\mathrm{s}}^{\prime t}} y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)-Q^t
\end{aligned} \tag{A.3}$$

Next, consider a supplier l verifying $y_l^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)<E_{l,\mathrm{s}}^t$. If $y_l^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=0$, it is known that $l\in\mathcal{N}_{\mathrm{s}}^t\setminus\mathcal{N}_{\mathrm{s}}^{\prime t}$ and $\mu_l^{t'}>\mu_j^{t'}$, for any supplier $j\in\mathcal{N}_{\mathrm{s}}^{\prime t}$. In this case, the upper bound in (A.1) can be raised up to the level $\sum_{j\in\mathcal{N}_{\mathrm{s}}^t} y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$. If $0<y_l^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)<E_{l,\mathrm{s}}^t$, then there are $l\in\mathcal{N}_{\mathrm{s}}^{\prime t}$ and $\mu_l^{t'}>\mu_j^{t'}$ for any supplier $j\in\mathcal{N}_{\mathrm{s}}^{\prime t}$. In this case, the upper bound in (A.3) is increased to $\sum_{j\in\mathcal{N}_{\mathrm{s}}^{\prime t}\setminus\{l\}} y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)$. In either case, there always exists a set of suppliers each submitting a reservation price lower than that of supplier l, who together can supply the total cleared quantities. So $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=0$ holds.

Lemma A.2: $\overline{p}_l^t(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-l}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\geqslant s_{-l}^t\left(\sum_{i\in\mathcal{N}_{\mathrm{d}}^{t(l)}}E_{i,\mathrm{d}}^t\right)$, for any supplier $l\in\mathcal{N}_{\mathrm{s}}^t$.

Proof of Lemma A.2: Since $s_{-l}^t(x)$ is a nondecreasing function, there are:

$$\begin{aligned}
& s_{-l}^t\left(\sum_{i\in\mathcal{N}_{\mathrm{d}}^{t(l)}}E_{i,\mathrm{d}}^t\right)\leqslant s_{-l}^t\left(\sum_{i\in\mathcal{N}_{\mathrm{d}}^t}x_i^{t(l)*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\right)\leqslant \\
& s_{-l}^t\left(\sum_{i\in\mathcal{N}_{\mathrm{d}}^t}x_i^{t(l)*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)+Q^t-E_{l,\mathrm{s}}^t\right)
\end{aligned} \tag{A.4}$$

Suppose that supplier l submits a reservation price $\mu_l^{t'} < s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^t} x_i^{t(l)*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t)+Q^t-E_{l,\mathrm{s}}^t\right)$. In this case, the solution to problem $\mathcal{P}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t)$ is the same as the solution to problem $\mathcal{P}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t)$ under the constraint $y_l^t = E_{l,\mathrm{s}}^t$. It then immediately follows that $y_l^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = y_l^{t(l)*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = E_{l,\mathrm{s}}^t$. From the definition of $\overline{p}_l^t$, it can be obtained that

$$\overline{p}_l^t(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-l}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) \geqslant s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^t} x_i^{t(l)*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t)+Q^t-E_{l,\mathrm{s}}^t\right) \tag{A.5}$$

Therefore, $s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right) \leqslant \overline{p}_l^t(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-l}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t)$ holds.

Lemma A.3: For any demander $k\in\mathcal{N}_\mathrm{d}^t$, $x_k^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-j}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_{-j,\mathrm{s}}^t) = E_{k,\mathrm{d}}^t$ is a solution to the problem $\mathcal{S}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t\setminus\{j\})$ for supplier $j\in\mathcal{N}_\mathrm{s}^t$.

Proof of Lemma A.3: If $k\in\mathcal{N}_\mathrm{d}^t$, it then holds that $x_k^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = E_{k,\mathrm{d}}^t$. From the monotonicity of the inverse supply function $s^t(q)$ and (22), there are:

$$\begin{aligned}\lambda_k^{t'} &\geqslant s^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^t} x_i^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t)+Q^t\right) \geqslant \\ & s^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^t} E_{i,\mathrm{d}}^t+Q^t\right) \geqslant s^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^t} E_{i,\mathrm{d}}^t+E_{j,\mathrm{d}}^t\right) \geqslant \\ & s_{-j}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^t\setminus\{k\}} x_i^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-j}^t,\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_{-j,\mathrm{s}}^t)+E_{k,\mathrm{d}}^t\right)\end{aligned} \tag{A.6}$$

Thus, $x_k^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-j}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_{-j,\mathrm{s}}^t) = E_{k,\mathrm{d}}^t$, otherwise the objective in the problem $\mathcal{S}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t\setminus\{j\})$ can be increased by increasing $x_k^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-j}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_{-j,\mathrm{s}}^t)$ to $E_{k,\mathrm{d}}^t$.

Based on the above three lemmas, it can be proved that it is a dominant strategy for each supplier to submit its true reservation price and give truthful quantity information. First, for any supplier l with a submission $\mu_l^{t'} > s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right)$, $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = 0$ holds. Consider a supplier l who submits a reservation price $\mu_l^{t'} > s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right)$. If $y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) < E_{l,\mathrm{s}}^t$, it follows from Lemma A.1 that $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = 0$. Otherwise, if $y_j^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = E_{l,\mathrm{s}}^t$, there is $\mathcal{N}_\mathrm{d}^t = \mathcal{N}_\mathrm{d}^{t(l)}$ and the total cleared quantity in the problem $\mathcal{S}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t)$ is $\sum_{i\in\mathcal{N}_\mathrm{d}^t} E_{i,\mathrm{d}}^t = \sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t$. Because $\mu_l^{t'} > s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right)$,, i.e., there exists a set of suppliers who can supply the total cleared quantities at reservation prices lower than that of supplier l, thus $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = 0$.

Second, suppose that supplier l submits $\mu_l^{t'} > s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right)$,. It follows from Lemma A.2 that $\mu_l^{t'} < \beta_l^t(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-l}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t)$ and $y_l^{t*}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = E_{l,\mathrm{s}}^t$. Thus, there is $\mathcal{N}_\mathrm{d}^t = \mathcal{N}_\mathrm{d}^{t(l)}$. Moreover, from Lemma A.2, it is known that $x_k^t = E_{k,d}^t$ in the solution to the problems $\mathcal{S}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t)$ and $\mathcal{S}(\mathcal{N}_\mathrm{d}^t,\mathcal{N}_\mathrm{s}^t\setminus\{l\})$, for any $k\in\mathcal{N}_\mathrm{d}^t = \mathcal{N}_\mathrm{d}^{t(l)}$. In addition, the two problems share the same total clearing quantity of $\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t$ and the difference in their solutions stems from the energy supplied by l. For $\mu_l^{t'} > s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right)$,, the following two cases are considered:

1) There exists a constant D, where $\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t - E_{l,\mathrm{s}}^t \leqslant D < \sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t$ such that $s_{-l}^t(D) < \mu_l^{t'} < s_{-l}^t(D+\varepsilon)$, for a sufficiently small $\varepsilon > 0$. In this case, $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = \sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t - D$ from optimality of the solution, and the revenue to supplier l is:

$$\begin{aligned} q_l^{t(\mathrm{D-CPA})} &= \mu_l^{t'} y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) + M(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) - \\ & M_{-l}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}_{-l}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_{-l,\mathrm{s}}^t) = \int_D^{\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,d}^t} s_{-l}^t\, q\mathrm{d}q \end{aligned} \tag{A.7}$$

2) $\mu_l^{t'} < s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t - E_{l,\mathrm{s}}^t\right)$. In this case, $y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_\mathrm{d}^t,\boldsymbol{E}_\mathrm{s}^t) = E_{l,\mathrm{s}}^t$ and the revenue to supplier l is:

$$q_l^{t(\mathrm{D-CPA})} = \int_{\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t - E_{l,s}^t}^{\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,d}^t} s_{-l}^t\, q\mathrm{d}q \tag{A.8}$$

According to the above two cases, it can be concluded that for a supplier l with $\mu_l^{t'} < s_{-l}^t\left(\sum_{i\in\mathcal{N}_\mathrm{d}^{t(l)}} E_{i,\mathrm{d}}^t\right)$, it sells either a part or all of its submitted quantity and each unit is paid by a nonincreasing price, from $s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t\right)$ to $s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t - y_l^{t**}(\boldsymbol{\lambda}^{t'},\boldsymbol{\mu}^{t'},\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)\right)$.

Third, it is a best response for every supplier to submit its true reservation price. Consider a supplier $l\in\mathcal{N}_s^t$, if its true reservation price μ_l^t satisfies $\mu_l^{t'} > s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t\right)$, then it prefers not to trade in the P2P market since each traded unit makes a nonpositive utility. The option of not trading can be achieved by submitting μ_l^t. If $\mu_l^{t'} < s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t - E_{l,s}^t\right)$, then supplier l prefers to sell all of its energy in the P2P market, which can also be achieved by submitting μ_l^t. If $s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t - E_{l,s}^t\right) \leqslant \mu_l^t \leqslant s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t\right)$, then supplier l prefers trading only a part of its supplied quantity of energy and submitting μ_l^t is the best response. Accordingly, it is the best for supplier l to submit its true reservation price under any circumstances.

Proof of Proposition 3: First, it is noted that any demander i will not overreport its maximum demand quantity since the unit utility $\gamma_i^t(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^{t'},\boldsymbol{E}_s^{t'})$ is nonincreasing with respect to $E_{i,d}^{t'}$.

Next, the conditions are identified under which demanders will not underreport their quantities. Suppose that the true reservation prices of prosumers and their submitted (demanded or supplied) quantities are independently distributed. Denote P^t as the reservation price of the marginal winner, whose energy allocation is Y, which is assumed to be randomly distributed according to $F(x)$, where $x\in\left[0,R^t\right]$. Let us consider a demander $i\in\mathcal{N}_d^t$ with reservation price λ_i^t and maximum demanded quantity $E_{i,d}^t$. If demander i submits $(\lambda_i^t, E_{i,d}^t-\varDelta)$, where $\varDelta>0$, it then receives an expected utility of:

$$\begin{aligned}&E_{P^t,P^{t'}}[Pr\left\{i\in\mathcal{N}_d^t\right\}(\lambda_i^t-P^t)(E_{i,d}^t-\varDelta)Pr\{\varDelta<Y\}+\\&\quad(\lambda_i^t-P^t)(E_{i,d}^t-\varDelta)Pr\{\varDelta>Y\}]=\\&\quad E_{P^t,P^{t'}}[Pr\left\{i\in\mathcal{N}_d^t\right\}(\lambda_i^t-P^t)(E_{i,d}^t-\varDelta)+\\&\quad Pr\left\{i\in\mathcal{N}_d^t\right\}(P^t-P^{t'})(E_{i,d}^t-\varDelta)F(\varDelta)]\end{aligned}\tag{A.9}$$

where $P^{t'}$ is the new marginal winner's reservation price when demander i submits $(E_{i,d}^t-\varDelta)$. The first term in (A.9) represents the utility reduction resulting from a decrease in the allocation of demander i, and the second term represents the utility increment resulting from a decrease in the buying price per unit. Note that $E\left[P^t-P^{t'}\right]\to 0$ when $N_d^t+N_s^t\to\infty$. Thus, the utility increment is negligible in the limit, and demander i will not underreport its quantity when the number of prosumers tends to infinity. In addition, since $E_{P^t,P^{t'}}\left[P^t-P^{t'}\right]\leqslant E_{P^t}[\lambda_i^t-P^t]$, there are:

$$\begin{aligned}&E_{P^t,P^{t'}}[Pr\left\{i\in\mathcal{N}_d^t\right\}(\lambda_i^t-P^t)(E_{i,d}^t-\varDelta)+\\&Pr\left\{i\in\mathcal{N}_d^t\right\}(P^t-P^{t'})(E_{i,d}^t-\varDelta)F(\varDelta)]\leqslant\\&E_{P^t}[Pr\left\{i\in\mathcal{N}_d^t\right\}(\lambda_i^t-P^t)(E_{i,d}^t-\varDelta)(1+F(\varDelta))]\end{aligned}\tag{A.10}$$

If $F(x)\leqslant\dfrac{x}{R^t-x}, x\in\left[0,\dfrac{R^t}{2}\right]$, it then holds that $1+F(\varDelta)\leqslant\dfrac{R^t}{R^t-\varDelta}\leqslant\dfrac{E_{i,d}^t}{E_{i,d}^t-\varDelta}$ for all $\varDelta\in\left(0,E_{i,d}^t\right]$. So, there is $E_{P_t}[Pr\left\{i\in\mathcal{N}_d^t\right\}(\lambda_i^t-P^t)(E_{i,d}^t-\varDelta)(1+F(\varDelta))]\leqslant E_{P_t}[Pr\left\{i\in\mathcal{N}_d^t\right\}(\lambda_i^t-P^t)E_{i,d}^t]$, which means demander i has no incentive to underreport its demand quantity.

Finally, it can be proved that suppliers will not underreport supplied quantities. To prove this conclusion, Supplier l's utility from submitting $E_{l,s}^{t'}$ is:

$$\int_{\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^{t'}-y_l^{t**}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^{t'},\boldsymbol{E}_s^{t'})}^{\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^{t'}}(s_{-l}^t q-\mu_l^t)\mathrm{d}q\tag{A.11}$$

Both $\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^{t'}$ and $y_l^{t**}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^{t'},\boldsymbol{E}_s^{t'})$ are nondecreasing in $E_{l,s}^{t'}$, and $s_{-l}^t(x)$ is increasing in x, thus supplier l's utility is nonincreasing when submitting a quantity $E_{l,s}^{t'}<E_{l,s}^t$.

Proof of Proposition 4: $x_i^{t*}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)<E_{i,d}^t$ if $\lambda_i^t<s^t\left(\sum_{i\in\mathcal{N}_d^t} x_i^{t*}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)+Q^t\right)$. So, the uniform buying price of all demanders satisfies $\alpha_i^t(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)\geqslant s^t\left(\sum_{i\in\mathcal{N}_d^t} E_{i,d}^t+Q^t\right)$. Next, we turn to the revenues to suppliers. From Lemma A.1, it is known that, for any supplier l with $y_l^{t**}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)>0$, $y_l^{t*}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_d^t,\boldsymbol{E}_s^t)=E_{l,s}^t$ holds. So there is $\mathcal{N}_d^{t(l)}=\mathcal{N}_d^t$, for any supplier l with a positive allocation in D-CPA. Moreover, the highest selling price for supplier l is:

$$s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^{t(l)}} E_{i,d}^t\right)=s_{-l}^t\left(\sum_{i\in\mathcal{N}_d^t} E_{i,d}^t\right)\leqslant s^t\left(\sum_{i\in\mathcal{N}_d^t} E_{i,d}^t+Q^t\right)\tag{A.12}$$

The preceding two inequalities, together with Proposition 1 and Lemma 1, lead to:

$$\sum_{i\in\mathcal{N}_{\mathrm{d}}^t} p_i^{t(\mathrm{D-CPA})}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\geqslant\sum_{i\in\mathcal{N}_{\mathrm{d}}^t} E_{i,\mathrm{d}}^t\alpha_i^t(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\geqslant$$
$$s^t\left(\sum_{i\in\mathcal{N}_{\mathrm{d}}^t}E_{i,\mathrm{d}}^t+Q^t\right)\sum_{i\in\mathcal{N}_{\mathrm{d}}^t}E_{i,\mathrm{d}}^t=$$
$$\left(\sum_{i\in\mathcal{N}_{\mathrm{d}}^t}E_{i,\mathrm{d}}^t+Q^t\right)\sum_{j\in\mathcal{N}_{\mathrm{s}}^t}y_j^{t**}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\geqslant$$
$$\sum_{j\in\mathcal{N}_{\mathrm{s}}^t}q_j^{t(\mathrm{D-CPA})}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t) \tag{A.13}$$

Therefore, the total payments from demanders are no less than the revenues to suppliers and D-CPA is subsidy-free.

Proof of Proposition 5: Suppose that the prosumers' truthful reservation prices are drawn randomly according to a distribution with support contained in $\left[0,a\right]$, and that the maximum demand quantity is bounded on $\left[0,R^t\right]$. Consider problem $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t,\mathcal{N}_{\mathrm{s}}^t\cup\{l\})$, which is problem $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t,\mathcal{N}_{\mathrm{s}}^t)$ with an additional supplier l selling Q^t units of energy at reservation price a. Since no demander has a reservation price higher than a and wants to trade with supplier l, it then immediately follows that $H(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)=H(\boldsymbol{\lambda}^t,(\boldsymbol{\mu}^t,a),\boldsymbol{E}_{\mathrm{d}}^t,(\boldsymbol{E}_{\mathrm{s}}^t,Q^t))$. And next, observe that $(\boldsymbol{x}^{t\wedge}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t),\boldsymbol{y}^{t\wedge}(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t),$ $y_l^t=Q)$ is a feasible solution of the problem $\mathcal{P}(\mathcal{N}_{\mathrm{d}}^t,\mathcal{N}_{\mathrm{s}}^t\cup\{l\})$, $H(\boldsymbol{\lambda}^t,(\boldsymbol{\mu}^t,a),\boldsymbol{E}_{\mathrm{d}}^t,(\boldsymbol{E}_{\mathrm{s}}^t,Q^t))\geqslant G(\boldsymbol{\lambda}^t,$ $\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)-aQ^t$ holds. Also, because at most one demander is allocated only a part of its demanded quantity, then:

$$H(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)-M(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\leqslant aR^t \tag{A.14}$$

Implying (A.15) as follows:

$$0\leqslant G(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)-M(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\leqslant a(Q^t+R^t) \tag{A.15}$$

There is:

$$0\leqslant\lim_{N_{\mathrm{d}}^t+N_{\mathrm{s}}^t\to\infty}\frac{G(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)-M(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)}{G(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)}\leqslant$$
$$\lim_{G(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)\to\infty}\frac{a(Q^t+R^t)}{G(\boldsymbol{\lambda}^t,\boldsymbol{\mu}^t,\boldsymbol{E}_{\mathrm{d}}^t,\boldsymbol{E}_{\mathrm{s}}^t)}=0 \tag{A.16}$$

ACKNOWLEDGMENT

Not applicable.

AUTHOR'S CONTRIBUTIONS

Shichang Cui and Shuang Xu: carried out the full-text writing and the construction of the paper framework. Fei Hu: helped in software and simulations. Yong Zhao, Jinyu Wen and Jinsong Wang: provided suggestions. All authors read and approved the final manuscript.

FUNDING

This work is supported by the National Natural Science Foundation of China (No. 52207108), and by the Science and Technology Project of State Grid Corporation of China (No. 1400202099523 A0000).

AVAILABILITY OF DATA AND MATERIALS

Please contact the corresponding author for data material request.

DECLARATIONS

Competing interests: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

AUTHORS' INFORMATION

Shichang Cui received the B.S. degree in automation and the Ph.D. degree in control science and engineering from Huazhong University of Science and Technology, Wuhan, China, in 2016 and 2021, respectively. He currently works as a postdoctoral fellow in the State Key Laboratory of Advanced Electromagnetic Engineering and Technology, the School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China. From Sep. 2019 to Sep. 2020, he was visiting the department of Mechanical Engineering, University of Victoria, Canada, supported by the CSC Joint Doctoral Program. His current research interests include stochastic optimization, distributed optimization, game theory, and energy management for smart grids.

Shuang Xu received the B.S. degree in mathematics from Hubei University of Education, Wuhan, China, in 2004 and the Ph.D. degree control science and engineering from Huazhong University of Science and Technology, Wuhan, China, in 2023. She currently works as a joint postdoctoral fellow in the State Key Laboratory of Advanced Electromagnetic Engineering and Technology, School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan, China and the State Grid Hubei Electrical Power Co., Ltd. Economics Research Institute, Wuhan, China. Her current research interests include game theory and power market.

Fei Hu received the B.S. degree in electrical engineering and automation from Southwest University, Chongqing, China, in 2018 and the M.S. degree in electrical engineering from Guangxi University, Guangxi, China, in 2021. He is currently pursuing a Ph.D. degree in artificial intelligence at Huazhong University of Science and Technology. His research interests include electricity markets, artificial intelligence, and power system optimization.

Yong Zhao received the Ph.D. degree in automation and system engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1996. He is currently a professor with the Huazhong University of Science and Technology. His current research interests include system engineering and artificial intelligence and its application in the smart grid. He was the deputy dean of Research Institute of Future Power Grid supported by State Grid Corporation of China and Huazhong University of Science and Technology.

Jinyu Wen received the B.S. and Ph.D. degrees in electrical engineering from the Huazhong University of Science and Technology (HUST), Wuhan, China, in 1992 and 1998, respectively. He was a visiting student from 1996 to 1997 and research fellow from 2002 to 2003 with the University of Liverpool, Liverpool, U.K., and a senior visiting researcher with the University of Texas at Arlington, Arlington, TX, USA, in 2010. From 1998 to 2002, he was a director engineer with XJ Electric Co. Ltd., China. In 2003, he joined the HUST. He is currently a professor with the School of Electrical and Electronics Engineering, HUST. His research interests include renewable energy integration, energy storage, multi-terminal HVDC, and power system operation and control.

Jinsong Wang received the B.S. degree in computer science from Kent State University, Kent, USA, in 2015. He is currently pursuing the Ph.D. degree in system and control engineering at Case Western Reserve University, Cleveland, USA. His research interests include artificial intelligence, load forecasting and fault diagnosis.

REFERENCES

[1] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era," *Nature Energy*, vol. 1, no. 4, pp. 1-6, 2016.

[2] Y. Zhou, J. Wu, and C. Long *et al*., "State-of-the-art analysis and perspectives for peer-to-peer energy trading," *Engineering,* vol. 6, no. 7, pp. 739-753, Jul. 2020.

[3] T. Morstyn, N. Farrell, and S. J. Darby *et al*., "Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants," *Nature Energy*, vol. 3, no. 2, pp. 94-101, 2018.

[4] H. Le Cadre, P. Jacquot, and C. Wan *et al*., "Peer-to-peer electricity market analysis: From variational to generalized Nnash equilibrium," *European Journal of Operational Research*, vol. 282, no. 2, pp. 753-771, Apr. 2020.

[5] T. Morstyn, A. Teytelboym, and M. D. McCulloch, "Bilateral contract networks for peer-to-peer energy trading," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2026-2035, Mar. 2019.

[6] Y. Liu, L. Wu, and J. Li, "Peer-to-peer (p2p) electricity trading in distribution systems of the future," *The Electricity Journal*, vol. 32, no. 4, pp. 2-6, May 2019.

[7] S. Chakraborty, T. Baarslag, and M. Kaisers, "Automated peer-to-peer negotiation for energy contract settlements in residential cooperatives," *Applied Energy*, vol. 259, Feb. 2020.

[8] C. Essayeh, M. R. El Fenni, and H. Dahmouni, "Optimization of energy exchange in microgrid networks: a coalition formation approach," *Protection and Control of Modern Power Systems*, vol. 4, no. 4, pp. 1-10, Dec. 2019.

[9] W. Tushar, T. K. Saha, and C. Yuen *et al*., "Peer-to-peer energy trading with sustainable user participation: A game theoretic approach," *IEEE Access*, vol. 6, pp. 62932-62943, Oct. 2018.

[10] C. Feng, F. Wen, and S. You *et al*., "Coalitional game-based transactive energy management in local energy communities," *IEEE Transactions on Power Systems*, vol. 35, no. 3, pp. 1729-1740, May 2020.

[11] J. Kang, R. Yu, and X. Huang *et al*., "Enabling localized peer-to-peer electricity trading among plug-in hybrid electric vehicles using consortium blockchains," *IEEE Transactions on Industrial Informatics*, vol. 13, no. 6, pp. 3154-3164, Dec. 2017.

[12] J. Guerrero, A. C. Chapman, and G. Verbic, "Decentralized p2p energy trading under network constraints in a low-voltage network," *IEEE Transactions on Smart Grid*, vol. 10, no. 5, pp. 5163-5173, Sept. 2019.

[13] A. Esmat, M. de Vos, and Y. Ghiassi-Farrokhfal *et al*., "A novel decentralized platform for peer-to-peer energy trading market with blockchain technology," *Applied Energy*, vol. 282, p. 116123, Jan. 2021.

[14] W. Tushar, B. Chai, and C. Yuen *et al*., "Energy storage sharing in smart grid: a modified auction-based approach," *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1462-1475, May 2016.

[15] W. Tushar, T. K. Saha, and C. Yuen *et al*., "Grid influenced peer-to-peer energy trading," *IEEE Transactions on Smart Grid*, vol. 11, no. 2, pp. 1407-1418, Mar. 2020.

[16] M. K. AlAshery, Z. Yi, and D. Shi *et al*., "A blockchain-enabled multi-settlement quasi-ideal peer-to-peer trading framework," *IEEE Transactions on Smart Grid*, vol. 12, no. 1, pp. 885-896, Jan. 2021.

[17] T. D. Nguyen and M. T. Thai, "A blockchain-based iterative double auction protocol using multiparty state channels," *ACM Transactions on Internet Technology*, vol. 21, no. 2, pp. 1-22, Mar. 2021.

[18] F. Ahsan, N. H. Dana, and S. K. Sarker *et al*., "Data-driven next-generation smart grid towards sustainable energy evolution: techniques and technology review," *Protection and Control of Modern Power Systems*, vol. 8, no. 3, pp. 1-42, Jul. 2023.

[19] J. Li, X. Liang, and D. Qu *et al*., "A pricing game for a photovoltaic prosumer cluster based on blockchain technology," *Power System Protection and Control*, vol. 52, no. 5, pp. 73-82, Mar. 2024. (in Chinese)

[20] N. Z. Aitzhan and D. Svetinovic, "Security and privacy in decentralized energy trading through multi-signatures, blockchain and anonymous messaging streams," *IEEE*

Transactions on Dependable and Secure Computing, vol. 15, no. 5, pp. 840-852, Sept. 2018.

[21] M. Faizan, T. Brenner, and F. Foerster *et al*., "Decentralized bottom-up energy trading using Ethereum as a platform," *Journal of Energy Markets*, vol. 12, no. 2, 2019.

[22] T. AlSkaif, J. L. Crespo-Vazquez, and M. Sekuloski *et al*., "Blockchain-based fully peer-to-peer energy trading strategies for residential energy systems," *IEEE Transactions on Industrial Informatics*, vol. 18, no. 1, pp. 231-241, Jan. 2022.

[23] J. Abdella, Z. Tari, and A. Anwar *et al*., "An architecture and performance evaluation of blockchain-based peer-to-peer energy trading," *IEEE Transactions on Smart Grid*, vol. 12, no. 4, pp. 3364-3378, Jul. 2021.

[24] Z. Li, J. Kang, and R. Yu *et al*., "Consortium blockchain for secure energy trading in industrial internet of things," *IEEE Transactions on Industrial Informatics*, vol. 14, no. 8, pp. 3690-3700, Aug. 2018.

[25] F. Luo, Z. Y. Dong, and G. Liang *et al*., "A distributed electricity trading system in active distribution networks based on multi-agent coalition and blockchain," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 4097-4108, Sept. 2019.

[26] P. Siano, G. De Marco, and A. Rol'an *et al*., "A survey and evaluation of the potentials of distributed ledger technology for peer-to-peer transactive energy exchanges in local energy markets," *IEEE Systems Journal*, vol. 13, no. 3, pp. 3454-3466, Sept. 2019.

[27] Z. Su, Y. Wang, and Q. Xu *et al*., "A secure charging scheme for electric vehicles with smart communities in energy blockchain," *IEEE Internet of Things Journal*, vol. 6, no. 3, pp. 4601-4613, Jun. 2019.

[28] D. Xiao, H. Chen, and W. Cai *et al*., "Integrated risk measurement and control for stochastic energy trading of a wind storage system in electricity markets," *Protection and Control of Modern Power Systems*, vol. 8, no. 4, pp. 1-11, Oct. 2023.

[29] N. Liu, X. Yu, and C. Wang *et al*., "Energy-sharing model with price-based demand response for microgrids of peer-to-peer prosumers," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 3569-3583, Sept. 2017.

[30] J. Zhao, B. Brereton, and M. Montalvo, "Gaming-based reserve constraint penalty factor analysis," *IEEE Transactions on Power Systems*, vol. 26, no. 2, pp. 616-626, May 2011.

[31] Y. Xu and S. H. Low, "An efficient and incentive compatible mechanism for wholesale electricity markets," *IEEE Transactions on Smart Grid*, vol. 8, no. 1, pp. 128-138, Jan. 2017.

[32] F. Lezama, J. Soares, and P. Hernandez-Leal *et al*., "Local energy markets: Paving the path toward fully transactive energy systems," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 4081-4088, Sept. 2019.

[33] L. Pereira, J. Cavaleiro, and L. Barros, "Economic assessment of solar-powered residential battery energy storage systems: The case of madeira island, Portugal," *Applied Sciences*, vol. 10, no. 20, p. 7366, Oct. 2020.

[34] H. Li, H. Hui, and H. Zhang, "Decentralized energy management of microgrid based on blockchain-empowered consensus algorithm with collusion prevention," *IEEE Transactions on Sustainable Energy*, vol. 14, no. 4, pp. 2260-2273, Oct. 2023.